2-5520 Theory of Mechanisms

Glossary

for bachelors study in 3rd year-classis, summer semester

Lecturer: Assoc. Prof. František Palčák, PhD., ÚAMM 02010

Lecture 6: Resulting velocity and resulting acceleration

Sections in Lecture 6:

S1 Carrying velocity

Carrying velocity

Let us apply the Poissont's decomposition of general motion 3/1 of the $\mathrm{PAR3}$ on fictive carrying motion 2/1, when $\mathrm{PAR3}$ is carried by $\mathrm{PAR2}$ $(3 \equiv 2)$.

The carrying velocity $\overline{\mathrm{v}}_{\mathrm{B21}}$ of the point $\mathrm{B} \in \mathrm{PAR2}$ is

$$\overline{\mathrm{v}}_{\mathrm{B21}} = \overline{\mathrm{v}}_{\mathrm{A21}} + \overline{\mathrm{v}}_{\mathrm{BA21}},$$

where

$$\overline{\mathrm{v}}_{\mathrm{BA21}} = \overline{w}_{21} \times \overline{\mathrm{r}}_{\mathrm{BA}},$$

so

$$\overline{\mathrm{v}}_{\mathrm{B21}} = \overline{\mathrm{v}}_{\mathrm{A21}} + \overline{w}_{21} \times \overline{\mathrm{r}}_{\mathrm{BA}} \tag{1}$$

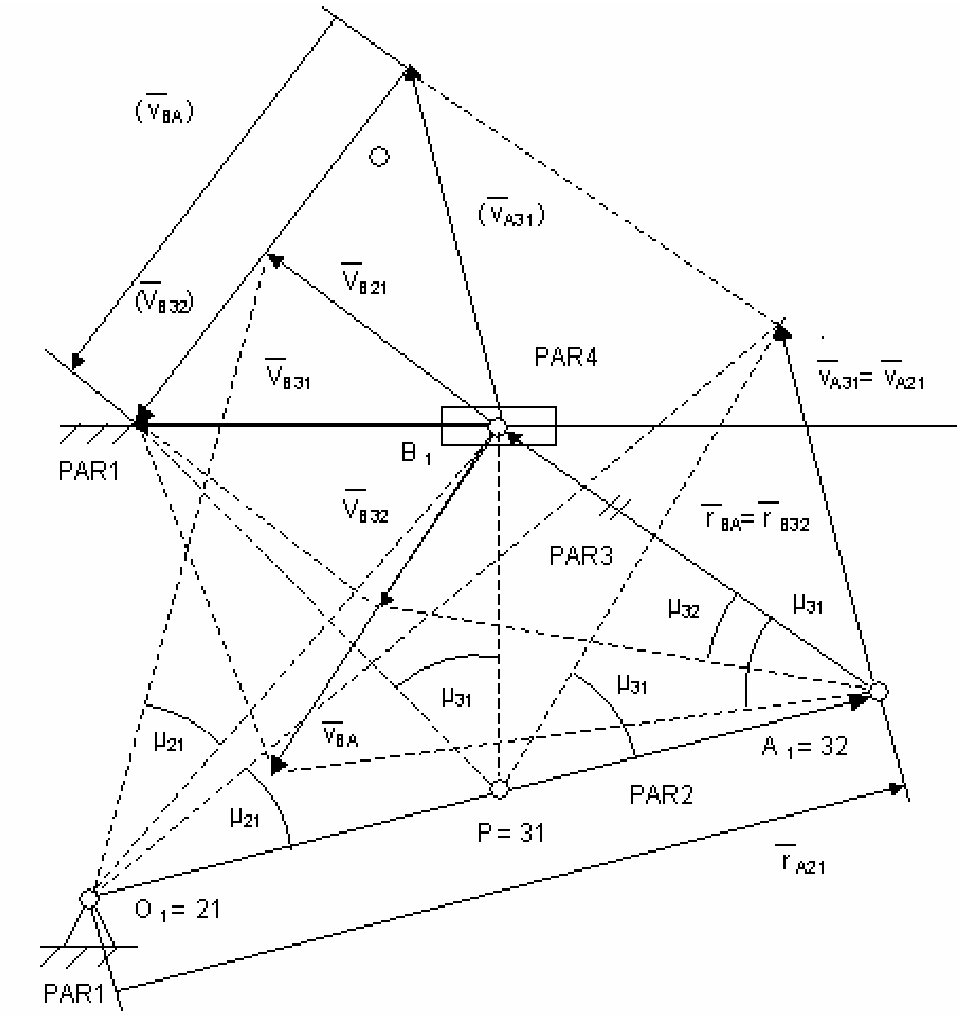

Fig.1 Graphical visualization of velocities during simultaneous motions

S2 Resulting velocity

Resulting velocity

Resulting velocity $\overline{v}_{B31}$ can be obtained by time derivative of position vector equation

$$\left[\overline{r}_{B31}\right]_1^{\bullet} = \left[\overline{r}_{A21}\right]_1^{\bullet} + \left[\overline{r}_{B32}\right]_1^{\bullet} \tag{2}$$

By direct time derivative we obtain

$$\left[\overline{r}_{B31}\right]_1^{\bullet} = \overline{v}_{B31} \tag{3}$$

$$\left[\overline{r}_{A21}\right]_1^{\bullet} = \overline{v}_{A21} \tag{4}$$

but for time derivative of $\overline{r}_{B32}$, expressed in the space $\{2\}$ in the space $\{1\}$ different as $\{2\}$ it is necessary apply a general rule

$$[\bar{r}_{B32}]_1^{\bullet} = [\bar{r}_{B32}]_2^{\bullet} + \bar{w}_{21} \times \bar{r}_{BA} \tag{5}$$

where $[\bar{r}_{P32}]_2^{\bullet} = \bar{v}_{B32}$ (6)

After substituting (3) - (6) into equation (2) we obtain vector equation for resulting velocity $\bar{v}_{B31}$ of the point B_{31}

$$\bar{v}_{B31} = \bar{v}_{B32} + \bar{v}_{B21} \tag{7}$$

which is sum of carrying velocity $\bar{v}_{B21}$ and local relative velocity $\bar{v}_{B32}$.

S3 Carrying acceleration

Carrying acceleration

If we want to obtain the carrying acceleration $\bar{a}_{B21}$ during fictive carrying motion 2/1, when $(3 \equiv 2)$, it is necessary derivative by time the equation (1) $\bar{v}_{B21} = \bar{v}_{A21} + \bar{w}_{21} \times \bar{r}_{BA}$

$$[\bar{v}_{B21}]_1^{\bullet} = [\bar{v}_{A21}]_1^{\bullet} + \bar{a}_{21} \times \bar{r}_{BA} + \bar{w}_{21} \times [\bar{r}_{BA21}]_1^{\bullet} \tag{8}$$

By direct time derivative we obtain

$$[\bar{v}_{B21}]_1^{\bullet} = \bar{a}_{B21} \tag{9}$$

$$[\bar{v}_{A21}]_1^{\bullet} = \bar{a}_{A21} \tag{10}$$

$$[\bar{r}_{BA21}]_1^{\bullet} = \bar{v}_{BA} = \bar{w}_{21} \times \bar{r}_{BA} \tag{11}$$

Then the final equation for carrying acceleration $\bar{a}_{B21}$ is

$$\bar{a}_{B21} = \bar{a}_{A21} + \bar{a}_{21} \times \bar{r}_{BA} + \bar{w}_{21} \times (\bar{w}_{21} \times \bar{r}_{BA}) \tag{12}$$

Let us denote symbolically three addends of carrying acceleration $\bar{a}_{B21}$ by numbered brackets

$$\bar{a}_{B21} = [\bar{a}_{A21}]_{(1)} + [\bar{a}_{21} \times \bar{r}_{BA}]_{(2)} + [\bar{w}_{21} \times (\bar{w}_{21} \times \bar{r}_{BA})]_{(3)}$$

S4 Resulting acceleration

Resulting acceleration

For purpose of the time derivative of equation (7) for velocities $\bar{v}_{B31} = \bar{v}_{B32} + \bar{v}_{B21}$ in the space $\{1\}$ we substitute term $\bar{v}_{B21}$ from decomposition of carrying motion 2/1 by equation (1) $\bar{v}_{B21} = \bar{v}_{A21} + \bar{w}_{21} \times \bar{r}_{BA}$, then

$$\overline{\mathrm{v}}_{\mathrm{B31}} = \overline{\mathrm{v}}_{\mathrm{B32}} + \overline{\mathrm{v}}_{\mathrm{A21}} + \overline{w}_{21} \times \overline{\mathrm{r}}_{\mathrm{BA}} \tag{13}$$

Now it is to derivative equation (13) by the time

$$\left[\overline{\mathrm{v}}_{\mathrm{B31}}\right]_1^{\bullet} = \left[\overline{\mathrm{v}}_{\mathrm{B32}}\right]_1^{\bullet} + \left[\overline{\mathrm{v}}_{\mathrm{A21}}\right]_1^{\bullet} + \left[\overline{w}_{21} \times \overline{\mathrm{r}}_{\mathrm{BA}}\right]_1^{\bullet} \tag{14}$$

By direct time derivative we obtain

$$\left[\overline{\mathrm{v}}_{\mathrm{B31}}\right]_1^{\bullet} = \overline{\mathrm{a}}_{\mathrm{B31}} \tag{15}$$

$$\left[\overline{\mathrm{v}}_{\mathrm{A21}}\right]_1^{\bullet} = \overline{\mathrm{a}}_{\mathrm{A21}} \tag{16}$$

Let us remind that by numbered bracket $\left[\overline{\mathrm{a}}_{\mathrm{A21}}\right]_{(1)}$ was denoted first addend of carrying acceleration $\overline{\mathrm{a}}_{\mathrm{B21}}$.
The time derivative of $\overline{\mathrm{v}}_{\mathrm{B32}}$ (expressed in the space $\{2\}$) in different space $\{1\}$ is

$$\left[\overline{\mathrm{v}}_{\mathrm{B32}}\right]_1^{\bullet} = \left[\overline{\mathrm{v}}_{\mathrm{B32}}\right]_2^{\bullet} + \overline{w}_{21} \times \overline{\mathrm{v}}_{\mathrm{B32}} \tag{17}$$

and by direct time derivative we obtain

$$\left[\overline{\mathrm{v}}_{\mathrm{B32}}\right]_1^{\bullet} = \overline{\mathrm{a}}_{\mathrm{B32}} \tag{18}$$

Time derivative of the last term from equation (14) is

$$\left[\overline{w}_{21} \times \overline{\mathrm{r}}_{\mathrm{BA}}\right]_1^{\bullet} = \overline{a}_{21} \times \overline{\mathrm{r}}_{\mathrm{BA}} + \overline{w}_{21} \times \left[\overline{\mathrm{r}}_{\mathrm{B32}}\right]_1^{\bullet} \tag{19}$$

The time derivative of $\overline{\mathrm{r}}_{\mathrm{B32}}$ (expressed in the space $\{2\}$) in different space $\{1\}$ is

$$\left[\overline{\mathrm{r}}_{\mathrm{B32}}\right]_1^{\bullet} = \left[\overline{\mathrm{r}}_{\mathrm{B32}}\right]_2^{\bullet} + \overline{w}_{21} \times \overline{\mathrm{r}}_{\mathrm{B32}} \tag{20}$$

again by direct time derivative we obtain

$$\left[\overline{\mathrm{r}}_{\mathrm{B32}}\right]_3^{\bullet} = \overline{\mathrm{v}}_{\mathrm{B32}} \tag{21}$$

After substituting (20), (21) into equation (19) we obtain

$$\left[\overline{w}_{21} \times \overline{\mathrm{r}}_{\mathrm{BA}}\right]_1^{\bullet} = \left[\overline{a}_{21} \times \overline{\mathrm{r}}_{\mathrm{BA}}\right]_{(2)} + \overline{w}_{21} \times \overline{\mathrm{v}}_{\mathrm{B32}} + \left[\overline{w}_{21} \times (\overline{w}_{21} \times \overline{\mathrm{r}}_{\mathrm{BA}})\right]_{(3)} \tag{22}$$

Let us remind that by numbered brackets $[\bar{a}_{21} \times \bar{\mathrm{r}}_{\mathrm{BA}}]_{(2)}$ and $[\bar{w}_{21} \times (\bar{w}_{21} \times \bar{\mathrm{r}}_{\mathrm{BA}})]_{(3)}$ were denoted addends of carrying acceleration $\bar{\mathrm{a}}_{\mathrm{B21}}$.

Then after substituting (15) - (21) into equation (14) the resulting acceleration $\bar{\mathrm{a}}_{\mathrm{B31}}$ is a sum

$$\bar{\mathrm{a}}_{\mathrm{B31}} = \bar{\mathrm{a}}_{\mathrm{B32}} + \bar{\mathrm{a}}_{\mathrm{B21}} + 2\bar{\omega}_{21} \times \bar{\mathrm{v}}_{\mathrm{B32}} \tag{23}$$

where the last term is sum of two terms $\bar{w}_{21} \times \bar{\mathrm{v}}_{\mathrm{B32}}$ from equations (14) and (22).

The term $2\bar{w}_{21} \times \bar{\mathrm{v}}_{\mathrm{B32}}$ in the equation (23) is known as Coriolis'es acceleration of the point B_{31}

$$\bar{\mathrm{a}}_{\mathrm{BCOR}} = 2\bar{w}_{21} \times \bar{\mathrm{v}}_{\mathrm{B32}} \tag{24}$$

The equation for resulting acceleration $\bar{\mathrm{a}}_{\mathrm{B31}}$ is

$$\bar{\mathrm{a}}_{\mathrm{B31}} = \bar{\mathrm{a}}_{\mathrm{B32}} + \bar{\mathrm{a}}_{\mathrm{B21}} + \bar{\mathrm{a}}_{\mathrm{BCOR}} \tag{25}$$

which is sum of carrying acceleration $\bar{\mathrm{a}}_{\mathrm{B21}}$, local relative acceleration $\bar{\mathrm{a}}_{\mathrm{B32}}$ and Coriolis'es acceleration $\bar{\mathrm{a}}_{\mathrm{BCOR}}$.