2-5520 Theory of Mechanisms

Glossary

for bachelors study in 3rd year-classis, summer semester

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Lecture 5: Velocities during simultaneous motions

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S1 Resulting angular velocity

Angular velocities

The position vector $\bar{r}_{B31}$ of the point B_{31} from PAR3 has components expressed in space $\{1\}$. Let us differentiate vector $\bar{r}_{B31}$ with respect to time in different spaces $\{2\}$, and $\{3\}$

$$[\bar{r}_{B31}]^{\bullet}_{1} = [\bar{r}_{B31}]^{\bullet}_{3} + \bar{w}_{31} \times \bar{r}_{B31} \tag{1}$$

$$[\bar{r}_{B31}]^{\bullet}_{1} = [\bar{r}_{B31}]^{\bullet}_{2} + \bar{w}_{21} \times \bar{r}_{B31} \tag{2}$$

$$[\bar{r}_{B31}]^{\bullet}_{2} = [\bar{r}_{B31}]^{\bullet}_{3} + \bar{w}_{32} \times \bar{r}_{B31} \tag{3}$$

Comparing (1), (2) and and substituting (3) we obtain

$$\bar{w}_{31} \times \bar{r}_{B31} = \bar{w}_{32} \times \bar{r}_{B31} + \bar{w}_{21} \times \bar{r}_{B31} \tag{4}$$

From equation (4) yield that resulting instantaneous angular velocity $\bar{w}_{31}$ of the of general planar motion 3/1 of the PAR3 (coupler in the piston–crank mechanism) wrt PAR1 in mechanism can be expressed as sum

$$\bar{w}_{31} = \bar{w}_{32} + \bar{w}_{21} \tag{5}$$

of instantaneous angular velocities $\bar{w}_{32}$, $\bar{w}_{21}$. It is the decomposition of general planar motion 3/1 of the PAR 3 wrt PAR1 (ground) when PAR3 is displaced from its initial position $\overrightarrow{A_1B_1}$ to the final position $\overrightarrow{A_2B_2}$ by fictive subsequent, or real simultaneous rotations 2/1 and 3/2. So general planar motion 3/1 of the PAR3 can be decomposed to the carrying motion 2/1 and local relative motion 3/2.

The instantaneous angular velocity $\bar{w}_{31}$ in the equation (5) we denote as the instantaneous resulting angular velocity and it is the sum of instantaneous local relative angular velocity $\bar{w}_{32}$ and instantaneous carrying angular velocity $\bar{w}_{21}$.

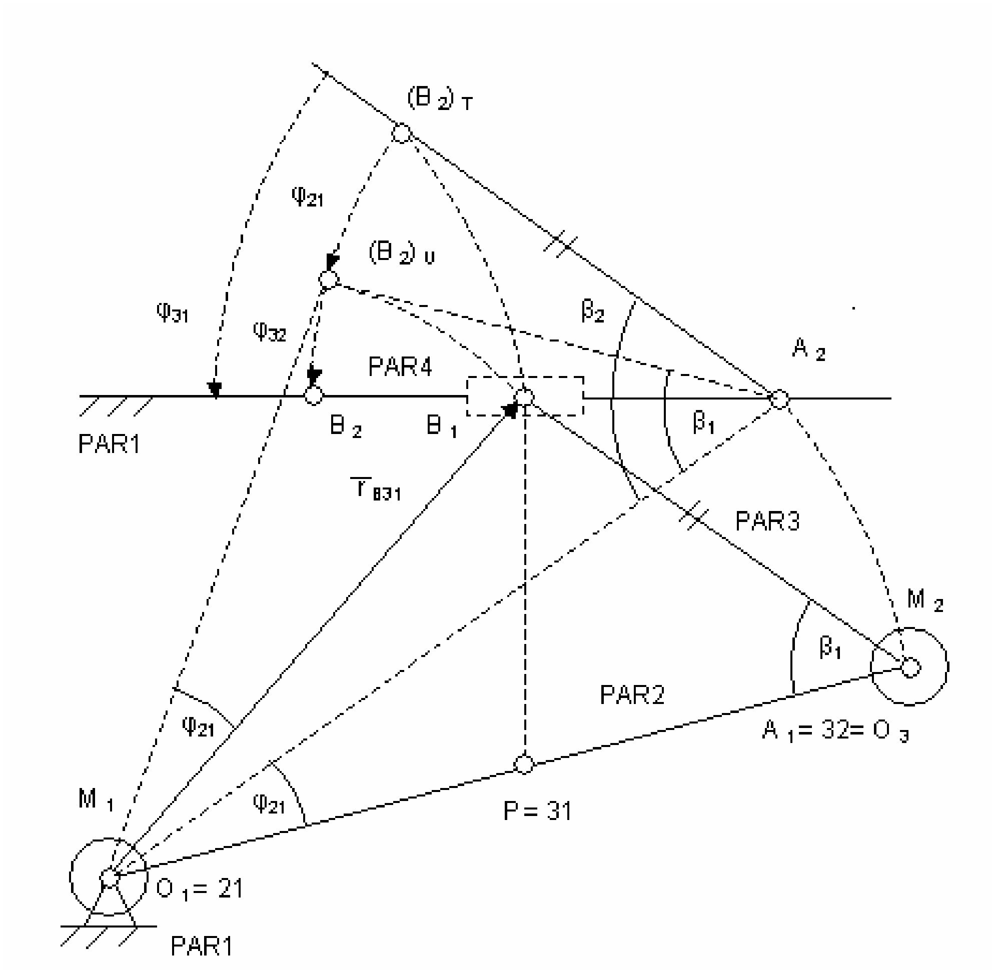

Fig.1 The open mechanism with arm PAR2 actuated by motor M1 and arm PAR3 actuated by motor M2.

S2 Simultaneous motions

Simultaneous motions

General plane motion 3/1 of the PAR3 can be decomposed to the fictive carrying motion 2/1 and fictive local relative motion 3/2.

Realization of this decomposition is demonstrated by open mechanism on Fig. 1 with arm PAR2 actuated by motor M1 and arm PAR3 actuated by motor M2. The PAR3 can be displaced from its initial position $\overrightarrow{A_1B_1}$ to the final position $\overrightarrow{A_2B_2}$ by subsequent, or simultaneous rotations 2/1 and 3/2.

During fictive carrying motion 2/1 about point O_1 by a finite angle φ_{21} the PAR3 is fixed to PAR2 $(3 \equiv 2)$ and PAR3 is displaced from its initial position $\overrightarrow{A_1B_1}$ to the the position $\overrightarrow{A_2(B_2)_U}$.
During fictive local relative rotation 3/2 the PAR3 is displaced about point O_3 by a finite angle φ_{32} from the position $\overrightarrow{A_2(B_2)_U}$ to the final position $\overrightarrow{A_2B_2}$.
For constant angular velocity $\omega = \text{const.}$ of rotation the slew angle is $\varphi = \omega t$ and considering the time $t = 1$ is also $\varphi = \omega$. Then resulting finite angle is the sum

$$\varphi_{31} = \varphi_{32} + \varphi_{21} \tag{6}$$

and corresponding fictive constant angular velocities, by which arms PAR2 and PAR3 in open mechanism rotates, are

$$\bar{\omega}_{31F} = \bar{\omega}_{32F} + \bar{\omega}_{21F} \tag{7}$$

The fictive constant angular velocities from Eq. (7) becomes actual angular velocities as are in the Eq. (5) for infinitesimal small angles in the Eq. (6).
The instantaneous angular velocity $\bar{\omega}_{31}$ in the equation (5) we denote as the instantaneous resulting angular velocity and it is the sum of instantaneous local relative angular velocity $\bar{\omega}_{32}$ and instantaneous carrying angular velocity $\bar{\omega}_{21}$.

S3 Resal's angular acceleration and resulting angular acceleration

Angular accelerations

Resulting angular acceleration $\bar{\alpha}_{31}$ we obtain via time derivative of equation (5)

$$[\bar{\omega}_{31}]_1^{\bullet} = [\bar{\omega}_{32}]_1^{\bullet} + [\bar{\omega}_{21}]_1^{\bullet} \tag{8}$$

By direct time derivative we obtain $[\bar{\omega}_{31}]_1^{\bullet} = \bar{\alpha}_{31}$ (9)
and time derivative of $\bar{\omega}_{32}$ in different space $\{1\}$ is

$$[\bar{\omega}_{32}]_1^{\bullet} = [\bar{\omega}_{32}]_2^{\bullet} + \bar{\omega}_{21} \times \bar{\omega}_{32} \tag{10}$$

where $[\bar{\omega}_{32}]_2^{\bullet} = \bar{\alpha}_{32}$ (11)

The cross product $\bar{\omega}_{21} \times \bar{\omega}_{32}$ is known as Resal's angular acceleration

$$\bar{a}_{\mathrm{R}} = \bar{w}_{21} \times \bar{w}_{32} \tag{12}$$

Substituting (9) – (12) into (8) we obtain equation for resulting angular acceleration

$$\bar{a}_{31} = \bar{a}_{32} + \bar{a}_{21} + \bar{a}_{\mathrm{R}} \tag{13}$$