2-5596 Mechanika viazaných mechanických systémov (VMS)
pre špecializáciu Aplikovaná mechanika, 4.roč. zimný sem.
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Pojmy pre vektorovú metódu kinematickej analýzy rovinnných mechanizmov

Kinematic analysis

Goal of kinematic analysis of the given planar mechanism with known initial values of all position coordinates of links and prescribed motion of input link/or links is to determine the time course for number $\mathrm{d}=2\mathrm{k}+\mathrm{s}_1$ of dependet global position coordinates, velocities and accelerations of output links.

Loop equations

The mathematical model with desired number d of lineary independet equations enabling to obtain number $\mathrm{d}=2\mathrm{k}+\mathrm{s}_1$ of dependet global position coordinates is necessary to develop according to type of mechanism

- for a planar closed mechanism (linkage) with revolute R and prismatic P joints, the number $\mathrm{d}=2\mathrm{k}$, ($\mathrm{s}_1=0$), (k is number of the basic kinematic loops in the structure of given planar mechanism) of unknown dependet global position coordinates can be determined from number $\mathrm{d}=2\mathrm{k}$ of explicit scalar loop constraint equations as projections of number k of vectorial loop equations into axes x_1, y_1

  $\mathrm{k}_\mathrm{j} \approx \sum_{\mathrm{i}=1}^{\mathrm{p_h}} \overline{\mathrm{h}}_\mathrm{i} = \overline{0}$, $\mathrm{j}=1,2,...,\mathrm{k}$, $\mathrm{i}=1,2,...,\mathrm{p_h}$

  where $\mathrm{p_h}$ is the number of oriented edges $\overline{\mathrm{h}}_\mathrm{i}$ in the polygon related to given planar mechanism.

- in a planar closed mechanism with correct slipping joint K of class $\mathrm{t}=1$ in which are mating adjacent links j and $\mathrm{j}+1$, it is necessary add to the number $\mathrm{d}=2\mathrm{k}$ of explicit scalar loop constraint equations one auxilliary explicit constraint equation

  $y_{\mathrm{1n(j+1)}} - y_{\mathrm{1n\,j}} = \mathrm{b}p$

  where $y_{\mathrm{1n\,j}} = y_{\mathrm{1j}} + y_{\mathrm{jp}} + n_\mathrm{j}$, and $y_{\mathrm{1j}} = \mathbf{S}(\mathrm{x}_1,\mathrm{x}_\mathrm{j})$ is global position coordinate, $y_{\mathrm{jp}} = \mathbf{S}(\mathrm{x}_\mathrm{j},\mathrm{p}_\mathrm{j})$, where p_j are lines of action for given radius vectors $\overline{\mathrm{r}}_\mathrm{j} = \mathrm{F}_\mathrm{j}(y_{\mathrm{jp}})$, which defines the shapes of contact surfaces, and $n_\mathrm{j} = \mathbf{S}(\overline{\mathrm{r}}_\mathrm{j},\overline{\mathrm{n}}_\mathrm{j})$ are angles between radius vectors $\overline{\mathrm{r}}_\mathrm{j}$ and outward normal vectors $\overline{\mathrm{n}}_\mathrm{j}$.

  Angles n_j are determined by the shapes of contact surfaces from equation $n_\mathrm{j} = \mathrm{arctg}_2(\mathrm{y/x})$, according to signum of projections y and x in the unit circle.

The coefficient b is resulting from initial mutual position of mating contact surfaces.

- In a planar closed mechanism the closed rolling joint V is of class $t = 2$, but singular configuration of adjacent links j and $j+1$ with mating circular shapes causes that rolling joint is incorrect, partially passive, with number $n_N = 1$ of uneliminated degrees of freedom. So actual mobility n_S of mechanism with number r_V of closed rolling joints is then
$n_S = n + r_V \; n_N$
where n is mobility of mechanism evaluated under assumption of its correctness.

- In a planar closed mechanism with open rolling joint V each open rolling joint V should be transformed into closed rolling joint imposing the auxilliary fictive binary link into mechanism. Each basic loop in mechanism with closed rolling joints of links with circular shapes degenerates into abscissa.
Because the number $d = 2k + s_1$ of dependet global position coordinates have to be determined, it is necessary add to the number $d = 2k$ of explicit scalar loop constraint equations one auxilliary explicit constraint equation of pure rolling condition resulting from basic equation of planetary (epicyclic) gear train
$R_C w_{1C} = (R_P + R_C) w_{1R} - R_P w_{1P}$
where
R_C is diameter of sun gear with angular velocity w_{1C},
R_R is length of arm (spider, or carrier) rotating with angular velocity w_{1R},
R_P is diameter of planet gear with angular velocity w_{1P}.

Example

It is to determine the time course of dependent global position coordinates of output links in the slider crank mechanism depicted on Fig.1.

The total number m of global relative position coordinates $y_i, i = 1,2,...,m$ of the links is a sum $m = n + d$ where n is number of independent global position coordinates of input links y_{ni}, $i = 1,2,...,n$, ($y_{n1} = y_{13}$ in our example) while n is mobility of mechanism and d is number of dependent global position coordinates of the links $y_{z\,i}$, $\mathrm{i}=1,2,...,\mathrm{d}$, ($y_{z1}=y_{13}$, $y_{z2}=\overline{\mathrm{p}}_{14}$ in our example).

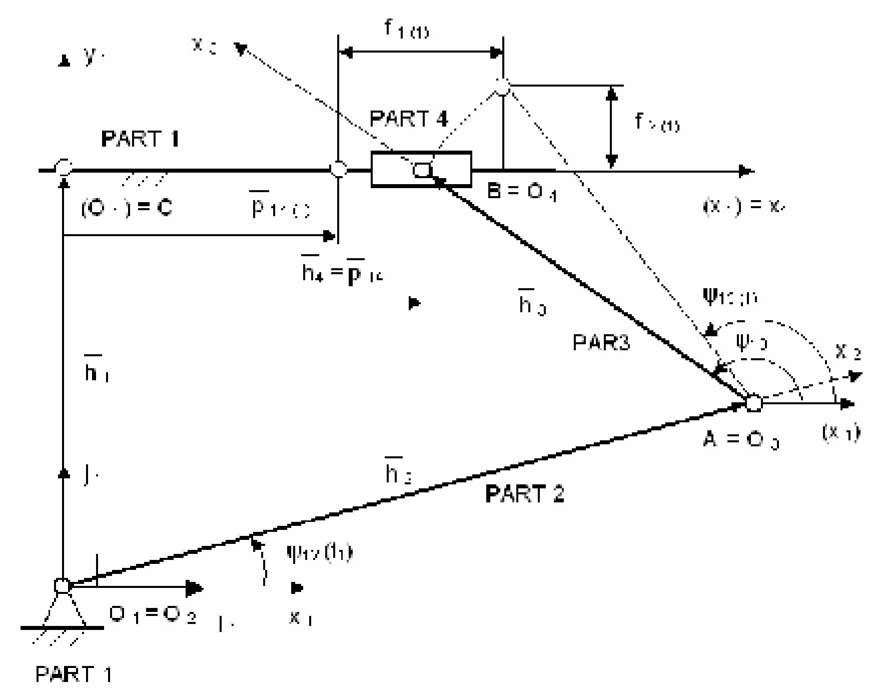

Fig.1 The slider crank mechanism in the initial configuration at the time $\mathrm{t}=\mathrm{t}_1$.

Loop equation

In our sigle loop slider crank mechanism from Fig.1 the closure condition of polygon is

$$\sum_{i=1}^{p_h} \overline{\mathrm{h}}_{\mathrm{i}} = \overline{0}, \ \mathrm{i}=1,2,...,\mathrm{p_h} \tag{1}$$

where $\mathrm{p_h}$ is the number of oriented edges $\overline{\mathrm{h}}_{\mathrm{i}}$ in the polygon related to given planar mechanism and $\overline{\mathrm{h}}_4=\overline{\mathrm{p}}_{14}$. Then vectorial loop equation according to direction of adding is

$$-\overline{\mathrm{h}}_1+\overline{\mathrm{h}}_2+\overline{\mathrm{h}}_3-\overline{\mathrm{p}}_{14}=\overline{0} \tag{2}$$

After multiplying by unit vectors $\overline{\mathrm{i}}_1,\overline{\mathrm{j}}_1$ we obtain scalar constraint equations as the projections of vectors into axes $\mathrm{x}_1,\mathrm{y}_1$

$$-\mathrm{h}_1 ca_1+\mathrm{h}_2 cy_{12}+\mathrm{h}_3 cy_{13}-\mathrm{p}_{14} ca_4=0 \tag{3}$$
$$-\mathrm{h}_1 sa_1+\mathrm{h}_2 sy_{12}+\mathrm{h}_3 sy_{13}-\mathrm{p}_{14} sa_4=0 \tag{4}$$

where $a_1=\mathbf{S}(\mathrm{x}_1,\mathrm{y}_1)$ and $a_4=\mathbf{S}(\mathrm{x}_1,\mathrm{x}_4)$

Constraint equations

Using arithmetic vector notation we can write scalar constraint equations (3), (4) in the form

$$\mathrm{f_i}(\bar{y}_{\mathrm{z}}) \approx \mathrm{f_i}(y_{\mathrm{z1}},...,y_{\mathrm{zd}}) = 0\,,\ \mathrm{i} = 1,2,...,\mathrm{d} \tag{5}$$

It is always possible to determine the time course for number d of unknown dependent global position coordinates y_{zi}, $\mathrm{i} = 1,2,...,\mathrm{d}$ of output links in the arithmetic vector $\bar{y}_{\mathrm{z}} = \lceil y_{\mathrm{z1}},\dots,y_{\mathrm{zd}} \rceil$ for known arithmetic vector $\bar{y}_{\mathrm{n}} = \lceil y_{\mathrm{n1}},\dots,y_{\mathrm{nn}} \rceil$ of independent global position coordinates of the input links, where n is mobility of mechanism, solving nonlinear algebraic constraint equations (5) by numerical Newton-Raphson (N-R) iteration method.

Newton-Raphson

The goal of the N-R method is to find the root $\bar{y}_{\mathrm{z}}$ of a function (5), that is to find new $\bar{y}_{\mathrm{z(2)}}$ such that $\bar{f}(\bar{y}_{\mathrm{z(2)}}) = \bar{0}$ if one knows the value of previous $\bar{y}_{\mathrm{z(1)}}$ for $\bar{f}(\bar{y}_{z(1)}) = \bar{0}$ and the value of all the partial derivatives $\dfrac{\partial \bar{f}(\bar{y}_{\mathrm{z(1)}})}{\partial \bar{y}_{\mathrm{z}}}$, $\dfrac{\partial^2 \bar{f}(\bar{y}_{\mathrm{z(1)}})}{\partial \bar{y}_{\mathrm{z}}^{\,2}}$, etc. Rather than computing all the derivatives, the series truncated after the first derivative and the approximation is repeated until the difference between two successive approximations is less than a small number e representing prescribed tolerance.

Assembly process

Assembly process is done in the initial configuration of mechanism (see Fig.1) when input independent global position coordinates at the time $\mathrm{t} = \mathrm{t}_1$ are constant.

Linearization

In the initial configuration of a mechanism the nonlinear algebraic constraint equations $\bar{f} = \lceil \mathrm{f}_1,...,\mathrm{f}_\mathrm{d} \rceil$ can be linearized by linear terms of Taylor series approximation in a summation with residuals $\bar{f}_{\mathrm{(r)}}$

$$\bar{f} \cong \bar{f}_{\mathrm{(r)}} + V_{\mathrm{(r)}} \Delta \bar{y}_{\mathrm{z\,(r)}} \tag{6}$$

where r is the number of iteration step,

Residuals

residuals $\bar{f}_{\mathrm{(r)}}$ are obtained after introduction of the arithmetic vector $\bar{y}_{\mathrm{z\,(r)}}$ of estimated position coordinates into constraint equations (5), matrix $V_{\mathrm{(r)}}$ is Jacobi matrix of the rank $(\mathrm{d\ x\ d})$

Jacobi matrix $V_{(r)} = \left[\frac{\partial f_i}{\partial y_{z\,j}} \right]_{(r)}$, $i = 1,2,...,m$, and $j = 1,2,...,d$ (7)

Corrections

and $\Delta \bar{y}_{z\,(r)}$ is unknown arithmetic vector of corrections. The arithmetic vector of nonlinear algebraic constraint equations is $\bar{f} = \bar{0}$ after introducing the exact solution $\bar{y}_z$. It stands to reason that residuals $\bar{f}_{(r)}$ are nonzero $\bar{f}_{(r)} \neq \bar{0}$ after introducing the arithmetic vector $\bar{y}_{z\,(r)}$ of estimated dependent global position coordinates into constraint equations (5), then also $\bar{f} \neq \bar{0}$. The arithmetic vector $\bar{y}_{z\,(r+1)}$ will be the accepted

Solution

solution, if the norm $\left\| \bar{f}_{(r)} \right\|$ of residuals satisfy condition $\left\| \bar{f}_{(r)} \right\| \leq e$, so maximum residual from all residuals is less than a specified tolerance e. This can be achieved by iteration process of converged (N-R) method

$$\bar{y}_{z\,(r+1)} = \bar{y}_{z\,(r)} + \Delta \bar{y}_{z\,(r)}, \quad r = 1,2,...,p \tag{8}$$

which will be finished, if the norm $\left\| \Delta \bar{y}_{z\,(r)} \right\|$ of corrections satisfy the condition

$$\left\| \Delta \bar{y}_{z\,(r)} \right\| \leq e \tag{9}$$

where e is prescribed tolerance. Then set $\bar{y}_{z\,(r+1)}$ of dependent global position coordinates satisfies all constraint equations (5).

In our example on Fig.1 the constraint equations (3),(4) are linearized according to Eq.(6)

$$f_1 \cong f_{1(1)} + \left[\frac{\partial f_1}{\partial y_{13}} \right]_{(1)} \Delta y_{13(1)} + \left[\frac{\partial f_1}{\partial p_{14}} \right]_{(1)} \Delta p_{14(1)} \tag{10}$$

$$f_2 \cong f_{2(1)} + \left[\frac{\partial f_2}{\partial y_{13}} \right]_{(1)} \Delta y_{13(1)} + \left[\frac{\partial f_2}{\partial p_{14}} \right]_{(1)} \Delta p_{14(1)} \tag{11}$$

To update dependent global position coordinates $y_{13(2)}$, $p_{14(2)}$ in second iteration step and improve initial estimation $y_{13(1)}$, $p_{14(1)}$, there are used corrections $\Delta y_{13(1)}$, $\Delta p_{14(1)}$ obtained solving linear equations (10), (11)

$$y_{13(2)} = y_{13(1)} + \Delta y_{13(1)} \quad (12)$$

$$p_{14(2)} = p_{14(1)} + \Delta p_{14(1)} \quad (13)$$

Kinematic analysis

The goal of the position-level kinematic analysis is to find $\bar{y}_z(t_2)$ for increment $\mathrm{in}\bar{y}_n(t_1) = \bar{y}_n(t_2) - \bar{y}_n(t_1)$ of input independent position coordinates (see Fig.2) corresponding to the defined time step $h = t_2 - t_1$. The vector $\bar{y}_z(t_1)$ obtained from assembly process at the time $t = t_1$ is now the estimate for N-R method.

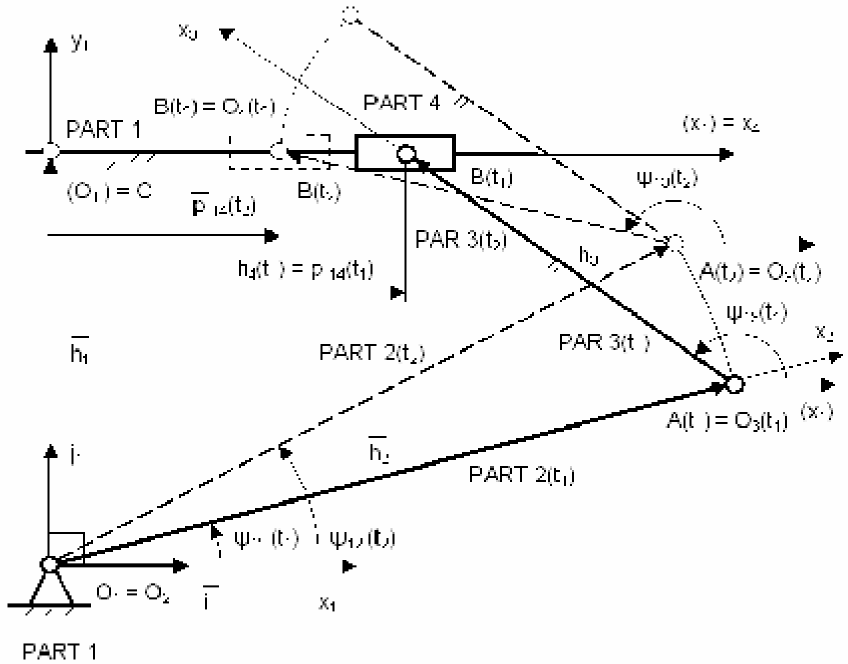

Fig.2 The slider crank mechanism configuration in the time steps t_1, t_2.

Position analysis

Goal of the position kinematic analysis of the given planar mechanism with known arithmetic vector $\bar{y}_n = \lceil y_{n1},...,y_{nn} \rceil$ of independet global position coordinates of the input link/or links is to determine the time course for number $d = 2k + s_1$ of unknown dependet global position coordinates of ouput links in the arithmetic vector $\bar{y}_z = \lceil y_{z1},...,y_{zd} \rceil$ solving nonlinear algebric constraint equations $f_i(y_{z1},...,y_{zd}) = 0$, $i = 1,2,...,d$ by numerical iteration Newton-Raphson (N-R) method.

In the initial configuration of mechanism the nonlinear algebric constraint equations $\bar{f}$ can be linearized by approximation with the sum of residual functions $\bar{f}_{(r)}$ obtained by introducing the arithmetic vector $\bar{y}_{z(r)}$ of estimated dependet global position coordinates (r is number of iteration step) and linear terms of Taylor series $\bar{f} \cong \bar{f}_{(r)} + V_{(r)} \Delta \bar{y}_{z(r)}$, where matrix $V_{(r)}$ is Jacobi matrix of the rank (d x d)

$V_{(r)} = \left[\dfrac{\partial f_i}{\partial y_{zj}} \right]_{(r)}$, $i = 1, 2, ..., m$, and $j = 1, 2, ..., d$, and $\Delta \bar{y}_{z(r)}$ is unknown arithmetic vector of corrections.

The arithmetic vector $\bar{f}$ of nonlinear algebric constraint equations is $\bar{f} = 0$ after introducing the solution $\bar{y}_z$, but $\bar{f} \neq \bar{f}_{(r)}$, $\bar{f}_{(r)} \neq 0$ (residuals) after introducing the arithmetic vector $\bar{y}_{z(r)}$ of estimated dependet global position coordinates. The arithmetic vector $\bar{y}_{z(r)}$ will be the accepted solution, if the norm $\left\| \bar{f}_{(r)} \right\|$ of residuals satisfy condition $\left\| \bar{f}_{(r)} \right\| \leq e$, so maximum residual from all residuals is less than a specified tolerance e. This can be achieved by iteration proces of converged numerical Newton-Raphson method $\bar{y}_{z(r+1)} = \bar{y}_{z(r)} + \Delta \bar{y}_{z(r)}$, which will be finished, if the norm $\left\| \Delta \bar{y}_{z(r)} \right\|$ of corrections satisfy the condition $\left\| \Delta \bar{y}_{z(r)} \right\| \leq e$ where e is prescribed tolerance.