Dynamics of point particle

1. *Three bodies with masses* $m_A = 10\,\mathrm{kg}$, $m_B = 15\,\mathrm{kg}$, $m_C = 20\,\mathrm{kg}$, *lying on a horizontal support and connected by a wire, are subject to a force* $F = 100\,\mathrm{N}$ *in the horizontal direction. The mass of the wire and the friction between the bodies and the support are negligible. Calculate the acceleration of the system and the force acting at each joint.*

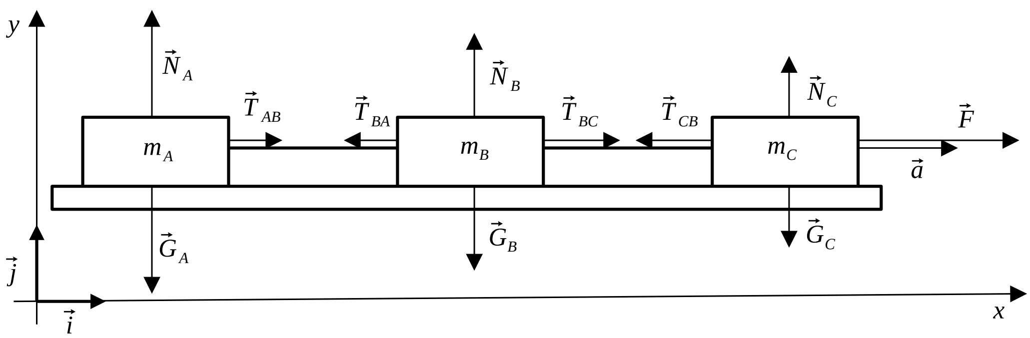

Fig. 1

The motion of bodies is described by Newton's law of force

$$\sum_{i=1}^{n} \vec{F}_i = m\vec{a}\ .$$

If the distance between the bodies does not change, the acceleration of all the bodies is equal

$$\vec{a}_A = \vec{a}_B = \vec{a}_C = \vec{a}\ .$$

The bodies are subject to gravitational forces $\vec{G}_A$, $\vec{G}_B$, $\vec{G}_C$, normal forces $\vec{N}_A$, $\vec{N}_B$, $\vec{N}_C$, the tensile forces of the wire $\vec{T}_{AB}$, $\vec{T}_{BA}$, $\vec{T}_{BC}$, $\vec{T}_{CB}$ and the external force $\vec{F}$, therefore Newton's force law will be

$$\vec{T}_{AB} + \vec{G}_A + \vec{N}_A = m_A\vec{a}\ ,$$

$$\vec{T}_{BA} + \vec{T}_{BC} + \vec{G}_B + \vec{N}_B = m_B\vec{a}\ ,$$

$$\vec{F} + \vec{T}_{CB} + \vec{G}_C + \vec{N}_C = m_C\vec{a}\ .$$

After scalar multiplication of the equations by the unit vectors $\vec{i}$ and $\vec{j}$ and using the equations

$$\vec{i}\cdot\vec{i} = 1\ ,$$

$$\vec{j} \cdot \vec{j} = 1 \; ,$$

$$\vec{i} \cdot \vec{j} = 0 \; ,$$

the equations take the scalar form

$$T_{AB} = m_A a \; ,$$

$$N_A - G_A = 0 \; ,$$

$$-T_{BA} + T_{BC} = m_B a \; ,$$

$$N_B - G_B = 0 \; ,$$

$$F - T_{CB} = m_C a \; ,$$

$$N_C - G_C = 0 \; .$$

In the vertical direction the acceleration is zero, therefore

$$N_A = G_A \; ,$$

$$N_B = G_B \; ,$$

$$N_C = G_C \; .$$

Newton's law of action and reaction implies

$$\vec{T}_{AB} = -\vec{T}_{BA} \; ,$$

$$T_{AB} = T_{BA} \; ,$$

$$\vec{T}_{BC} = -\vec{T}_{CB} \; ,$$

$$T_{BC} = T_{CB} \; .$$

Therefore, in the horizontal direction

$$T_{AB} = m_A a \; ,$$

$$-T_{AB} + T_{BC} = m_B a \; ,$$

$$F - T_{BC} = m_C a \; .$$

By summing all the equations, the acceleration of the system can be expressed

$$a = \frac{F}{m_A + m_B + m_C}$$

and then modifying the individual equations of the force between the bodies

$$T_{AB} = \frac{F m_A}{m_A + m_B + m_C} \ ,$$

$$T_{BC} = \frac{F (m_A + m_B)}{m_A + m_B + m_C} \ .$$

After substituting the values, the numerical solution will be

$$a = \frac{100\,\mathrm{N}}{10\,\mathrm{kg} + 15\,\mathrm{kg} + 20\,\mathrm{kg}} = 2{,}2\,\mathrm{m\,s^{-2}} \ ,$$

$$T_{AB} = \frac{100\,\mathrm{N}\ .\ 10\,\mathrm{kg}}{10\,\mathrm{kg} + 15\,\mathrm{kg} + 20\,\mathrm{kg}} = 22{,}2\,\mathrm{N} \ ,$$

$$T_{BC} = \frac{100\,\mathrm{N}\ .\ (10\,\mathrm{kg} + 15\,\mathrm{kg})}{10\,\mathrm{kg} + 15\,\mathrm{kg} + 20\,\mathrm{kg}} = 55{,}6\,\mathrm{N} \ .$$

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2. *Two bodies with equal masses $m = 5\,\mathrm{kg}$ are connected by a wire passing through a freely rotating pulley. The first body hangs freely on the wire, the second lies on an inclined plane which makes an angle $\alpha = 30^\circ$ with the horizontal plane. Calculate the acceleration of the bodies and the force acting at the wire if there is no friction between the body and the inclined plane and if there is friction between the body and the inclined plane and the friction factor is $\mu = 0{,}2$.*

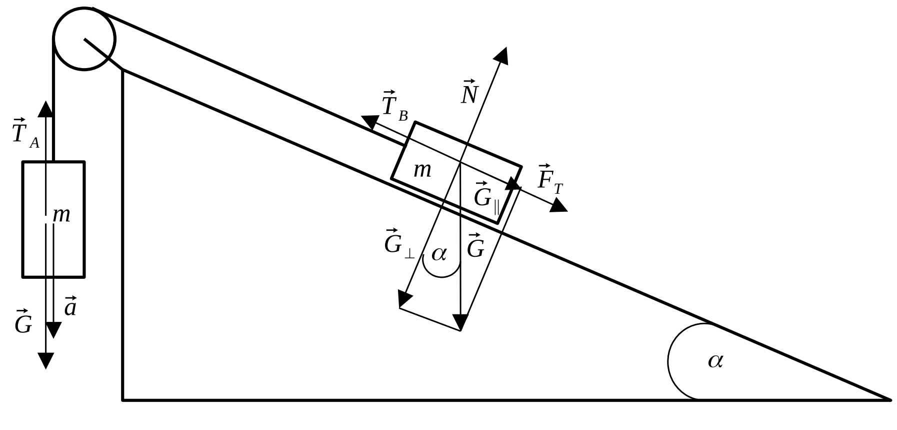

Fig. 2

The gravitational force of a body on an inclined plane can be decomposed into a component parallel to the inclined plane and a component perpendicular to the inclined plane

$$\vec{G} = \vec{G}_\parallel + \vec{G}_\perp \ .$$

The component parallel to the inclined plane will have the magnitude

$$G_{\parallel} = G \sin \alpha$$

and the component perpendicular to the inclined plane

$$G_{\perp} = G \cos \alpha \ .$$

The magnitude of the frictional force between the body and the inclined plane is

$$F_T = \mu G_{\perp} = \mu G \cos \alpha \ .$$

Newton's law of force

$$\sum_{i=1}^{n} \vec{F}_i = m\vec{a} \ ,$$

for a hanging body is

$$\vec{G} + \vec{T}_A = m\vec{a}_A$$

and for a body on an inclined plane is

$$\vec{G}_{\parallel} + \vec{G}_{\perp} + \vec{N} + \vec{T}_B + \vec{F}_T = m\vec{a}_B \ .$$

Newton's law of action and reaction implies

$$T_A = T_B = T \ .$$

The length of the wire does not change, therefore,

$$a_A = a_B = a \ .$$

The force of gravity can be calculated using the acceleration of gravity as

$$\vec{G} = m\vec{g} \ .$$

Newton's law of force for bodies has a scalar form

$$mg - T = ma \ ,$$

$$-mg \sin \alpha + T - \mu mg \cos \alpha = ma \ .$$

By solving the system of equations, it is possible to express the acceleration of the system

$$a = \frac{g(1 - \sin \alpha - \mu \cos \alpha)}{2} \ ,$$

and the force acting on the wire

$$mg - T = ma \implies T = mg - ma \ ,$$

$$T = \frac{mg(1 + \sin\alpha + \mu\cos\alpha)}{2} \ .$$

If the friction between the body and the inclined plane is negligible

$$\mu = 0 \ ,$$

the acceleration of the system will be

$$a = \frac{g(1 - \sin\alpha)}{2}$$

and the force acting on the wire will be

$$T = \frac{mg(1 + \sin\alpha)}{2} \ .$$

After substituting the numerical values

$$a = \frac{9{,}81\,\mathrm{m\,s^{-2}} \ . \ (1 - \sin 30^\circ)}{2} = 2{,}45\,\mathrm{m\,s^{-2}} \ ,$$

$$T = \frac{5\,\mathrm{kg} \ . \ 9{,}81\,\mathrm{m\,s^{-2}} \ . \ (1 + \sin 30^\circ)}{2} = 36{,}79\,\mathrm{N} \ .$$

If the friction between the body and the inclined plane is not negligible

$$\mu = 0{,}1 \ ,$$

the acceleration of the system will be

$$a = \frac{g(1 - \sin\alpha - \mu\cos\alpha)}{2}$$

and the force acting on the wire will be

$$T = \frac{mg(1 + \sin\alpha + \mu\cos\alpha)}{2} \ .$$

After substituting the numerical values

$$a = \frac{9{,}81\,\mathrm{m\,s^{-2}} \ . \ (1 - \sin 30^\circ - 0{,}2 \ . \ \cos 30^\circ)}{2} = 1{,}60\,\mathrm{m\,s^{-2}} \ ,$$

$$T = \frac{5\,\mathrm{kg} \ . \ 9{,}81\,\mathrm{m\,s^{-2}} \ . \ (1 + \sin 30^\circ + 0{,}2 \ . \ \cos 30^\circ)}{2} = 41{,}04\,\mathrm{N} \ .$$

3. *A weight of mass $m = 5\,\mathrm{kg}$ hanging on a wire of length $l = 1\,\mathrm{m}$ swings with a maximum angular deflection $\alpha = 60°$. What force F_1 is acting on the wire at the extreme positions and what force F_2 is acting on the wire at the lowest position?*

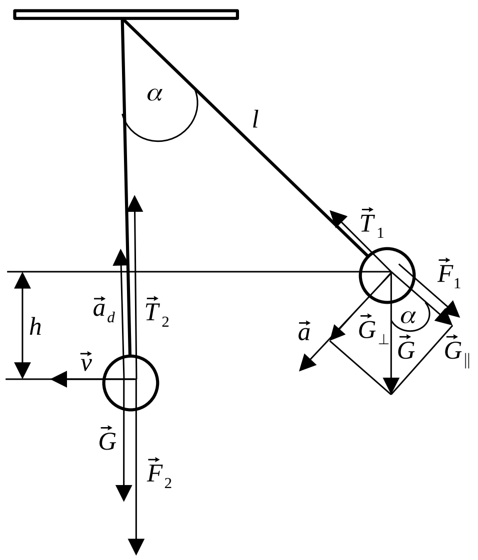

Fig. 3

The weight in the extreme position is acted upon by the gravitational force $\vec{G}$ and the force of the wire $\vec{T}_1$. Newton's law of force will therefore have the form

$$\vec{G} + \vec{T}_1 = m\vec{a} \; .$$

The gravitational force can be resolved into a component parallel to the direction of the wire and a component perpendicular to the direction of the wire

$$\vec{G} = \vec{G}_{\|} + \vec{G}_{\perp} \; .$$

The component of the gravitational force acting in the direction parallel to the direction of the wire is

$$G_{\|} = G \cos\alpha \; ,$$

from Newton's law of forces for components in the direction of the wire implies

$$G_{\|} - T_1 = 0 \; ,$$

$$mg \cos\alpha - T_1 = 0 \; ,$$

from which it is possible to express the force which the wire acts on the weight

$$T_1 = mg\cos\alpha \ .$$

According to Newton's law of action and reaction, the force which the weight acts on the wire is equal in magnitude and in the opposite direction to the force which the wire acts on the weight

$$\vec{F}_1 = -\vec{T}_1 \ ,$$

therefore, the magnitude of the force that acts on the wire at its extreme position will be

$$F_1 = mg\cos\alpha \ ,$$

after substituting numerical values

$$F_1 = 5\,\mathrm{kg}\ .\ 9{,}81\,\mathrm{m\,s^{-2}}\ .\ \cos 60^\circ = 24{,}53\,\mathrm{N}\ .$$

The height of the weight at its extreme position relative to the lowest position can be expressed as

$$h = l - l\cos\alpha \ .$$

The law of conservation of mechanical energy for a weight in the extreme and lowest position has the form

$$E_{p1} + E_{k1} = E_{p2} + E_{k2} \ ,$$

if in the extreme position the kinetic energy is zero

$$E_{k1} = 0$$

and in the lowest position the potential energy is zero

$$E_{p2} = 0 \ ,$$

the law of conservation of mechanical energy takes the form

$$E_{p1} = E_{k2} \ ,$$

$$mgh = \frac{mv^2}{2} \ ,$$

from which it is possible to express the velocity of the weight in the lowest position as

$$v = \sqrt{2gh} \; .$$

The weight will move in a circle with radius l and will be acted upon by the gravitational force $\vec{G}$ and the force of the wire $\vec{T}_2$. Newton's law of force for the body at its lowest position will be

$$\vec{G} + \vec{T}_2 = m\vec{a}_d \; ,$$

at lowest position in the direction of the wire, it states

$$mg - T_2 = -ma_d \; .$$

The magnitude of the centripetal acceleration at its lowest position can be expressed using the velocity of the weight

$$a_d = \frac{v^2}{R} = \frac{2gh}{l} = 2g(1 - \cos\alpha) \; .$$

The force stretching the wire at its lowest point will be

$$T_2 = mg + ma_d = mg(3 - 2\cos\alpha) \; .$$

The force exerted by the weight on the wire is equal in magnitude and opposite in direction to the force exerted by the wire on the weight.

$$\vec{F}_2 = -\vec{T}_2 \; ,$$

therefore, the magnitude of the force that acts on the wire in the lowest position will be

$$F_2 = mg(3 - 2\cos\alpha) \; ,$$

after substituting numerical values

$$F_1 = 5\,\mathrm{kg} \; . \; 9{,}81\,\mathrm{m\,s^{-2}} \; . \; (3 - 2 \; . \; \cos 60°) = 98{,}1\,\mathrm{N} \; .$$

4. *What impulse will the wall give to an elastic ball with mass* $m = 1\,\mathrm{kg}$ *and velocity* $v_0 = 10\,\mathrm{ms}^{-1}$ *that hits the wall in a direction making an angle* $\alpha = 60°$ *with the normal?*

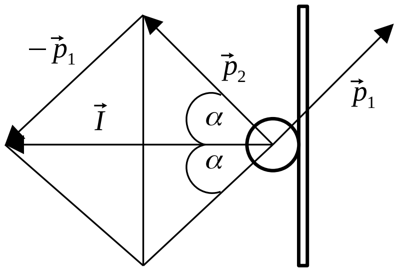

Fig. 4

By definition, the impulse of a force expresses the time effect of a force

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}(t)\, dt \ ,$$

to calculate it, the impulse theorem can also be used

$$\vec{I} = \vec{p}_2 - \vec{p}_1 \ .$$

In an elastic collision, the kinetic energy of the ball does not change

$$E_{k2} = E_{k1} \ ,$$

$$\frac{mv_2^2}{2} = \frac{mv_1^2}{2} \ ,$$

therefore the magnitude of the velocity does not change

$$v_2 = v_1 = v$$

and therefore the magnitude of the momentum does not change either

$$p_2 = p_1 = p = mv \ .$$

From the figure (Fig. 5) it follows

$$\cos\alpha = \frac{I}{2p} \ ,$$

from which it is possible to express the impulse of the force

$$I = 2p\cos\alpha = 2mv\cos\alpha \ ,$$

after substituting numerical values

$$I = 2 \ . \ 1\,\mathrm{kg} \ . \ 10\,\mathrm{ms}^{-1} \ . \ \cos 60° = 10\,\mathrm{N\,s} \ .$$

5. *A body lying on a horizontal surface is acted upon in the horizontal direction by a force whose time dependence is* $F(t) = A + Bt + Ct^2$*, where* $A = 0{,}2\,\mathrm{N}$*,* $B = 0{,}4\,\mathrm{N\,s^{-1}}$*,* $C = 0{,}6\,\mathrm{N\,s^{-2}}$*. Calculate the impulse of the force for the time* $t_0 = 5\,\mathrm{s}$*?*

The impulse of a force expresses the time effect of a force

$$\vec{I} = \int_0^{t_0} \vec{F}(t)\,dt\ .$$

In the case of linear motion, the magnitude of the impulse of a force will be

$$I = \int_0^{t_0} F(t)\,dt = \int_0^{t_0} (A+Bt+Ct^2)\,dt = \left[At + B\frac{t^2}{2} + C\frac{t^3}{3}\right]_0^{t_0} = At_0 + B\frac{t_0^2}{2} + C\frac{t_0^3}{3}\ .$$

Substituting numerical values

$$I = 0{,}2\,\mathrm{N}\ .\ 5\,\mathrm{s} + 0{,}4\,\mathrm{N\,s^{-1}}\ .\ \frac{(5\,\mathrm{s})^2}{2} + 0{,}6\,\mathrm{N\,s^{-2}}\ .\ \frac{(5\,\mathrm{s})^3}{3} = 31\,\mathrm{N\,s}\ .$$

6. *A point particle of mass* $m = 5\,\mathrm{kg}$ *is moved by a force such that its path changes with time as* $x(t) = A + Bt + Ct^2 + Dt^3$*, where* $C = 2\,\mathrm{m\,s^{-2}}$*,* $D = -0{,}2\,\mathrm{m\,s^{-3}}$*. Calculate the magnitude of the force acting on the mass point at time* $t_0 = 2\,\mathrm{s}$ *and find the time when the force will be zero.*

Velocity in a linear motion can be expressed from its definition as

$$v(t) = \frac{\mathrm{d}x(t)}{\mathrm{d}t} = \frac{\mathrm{d}(A + Bt + Ct^2 + Dt^3)}{\mathrm{d}t} = B + 2Ct + 3Dt^2\ .$$

Acceleration in a linear motion can be expressed from its definition as

$$a(t) = \frac{\mathrm{d}v(t)}{\mathrm{d}t} = \frac{\mathrm{d}(B + 2Ct + 3Dt^2)}{\mathrm{d}t} = 2C + 6Dt\ .$$

The force can be calculated from Newton's law of force

$$F(t) = ma(t) = m(2C + 6Dt)\ .$$

At time t_0 the force will be

$$F(t = t_0) = m(2C + 6Dt_0)\ .$$

After substituting numerical values

$$F(t = 2\,\mathrm{s}) = 5\,\mathrm{kg}\ .\ \left[2\ .\ 2\,\mathrm{m\,s^{-2}} + 6\ .\ (-0{,}2\,\mathrm{m\,s^{-3}})\ .\ 2\,\mathrm{s}\right] = 8\,\mathrm{N}\ .$$

From the condition that the force is zero

$$F(t) = 0 \, ,$$

it follows

$$m(2C + 6Dt) = 0 \, ,$$

from which it is possible to express the time when the force will be zero

$$t = \frac{-C}{3D}$$

and after substituting numerical values

$$t = \frac{-2\,\mathrm{m\,s^{-2}}}{3 \,.\, (-0{,}2\,\mathrm{m\,s^{-3}})} = 3{,}33\,\mathrm{s} \, .$$

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7. *A force $F = F_0 - kt$ acts on a body of mass $m = 5\,\mathrm{kg}$, where $F_0 = 10\,\mathrm{N}$ and $k = 0{,}1\,\mathrm{N\,s^{-1}}$ are constants. Express the acceleration, velocity, and position of the body at time $t = 10\,\mathrm{s}$ if the body initially had a velocity $v_0 = 2\,\mathrm{m\,s^{-1}}$ and a the starting position was $x_0 = 2\,\mathrm{m}$.*

From Newton's law of force

$$\vec{F} = m\vec{a} \, ,$$

for the acceleration of a body in linear motion follows

$$a = \frac{F}{m} = \frac{F_0}{m} - \frac{kt}{m} \, ,$$

after substituting numerical values, the acceleration at time $t = 10\,\mathrm{s}$ will be

$$a = \frac{10\,\mathrm{N}}{5\,\mathrm{kg}} - \frac{0{,}1\,\mathrm{N\,s^{-1}} \,.\, 10\,\mathrm{s}}{5\,\mathrm{kg}} = 2{,}2\,\mathrm{m\,s^{-2}} \, .$$

The velocity of a point particle can be calculated using the acceleration

$$v = \int a\,dt = \int \left(\frac{F_0}{m} - \frac{kt}{m} \right) dt = \frac{F_0 t}{m} + \frac{kt^2}{2m} + c_1 \, .$$

If the initial velocity was v_0, the integration constant c_1 will be

$$v(t = 0\,\mathrm{s}) = v_0 \implies c_1 = v_0$$

and the velocity of the point particle will be

$$v = \frac{F_0 t}{m} + \frac{kt^2}{2m} + v_0 \ ,$$

after substituting numerical values, the velocity at time $t = 10\,\mathrm{s}$ will be

$$v = \frac{10\,\mathrm{N}\ .\ 10\,\mathrm{s}}{5\,\mathrm{kg}} + \frac{0{,}1\,\mathrm{N\,s^{-1}}\ .\ (10\,\mathrm{s})^2}{2\ .\ 5\,\mathrm{kg}} + 2\,\mathrm{m\,s^{-1}} = 23\,\mathrm{m\,s^{-1}}\ .$$

The position of a mass point can be calculated using the velocity

$$x = \int v\,dt = \int \left(\frac{F_0 t}{m} - \frac{kt^2}{2m} + v_0 \right) dt = \frac{F_0 t^2}{2m} - \frac{kt^3}{6m} + v_0 t + c_2 \ ,$$

if the initial position was x_0, the integration constant c_2 will be

$$x(t = 0\,\mathrm{s}) = x_0 \implies c_2 = x_0$$

and the position of the mass point will be

$$x = \frac{F_0 t^2}{2m} - \frac{kt^3}{6m} + v_0 t + x_0 \ ,$$

after substituting numerical values, the position at time $t = 10\,\mathrm{s}$ will be

$$x = \frac{10\,\mathrm{N}\ .\ (10\,\mathrm{s})^2}{2\ .\ 5\,\mathrm{kg}} + \frac{0{,}1\,\mathrm{N\,s^{-1}}\ .\ (10\,\mathrm{s})^3}{6\ .\ 5\,\mathrm{kg}} + 2\,\mathrm{m\,s^{-1}}\ .\ 10\,\mathrm{s} + 2\,\mathrm{m} = 125{,}33\,\mathrm{m}\ .$$

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8. *What work must be done to compress the buffer spring of a wagon by* $x_0 = 10\,\mathrm{cm}$, *when a force of* $F_1 = 25\,000\,\mathrm{N}$ *is required to compress it by* $x_1 = 1\,\mathrm{cm}$ *and the force is directly proportional to the shortening of the spiral.*

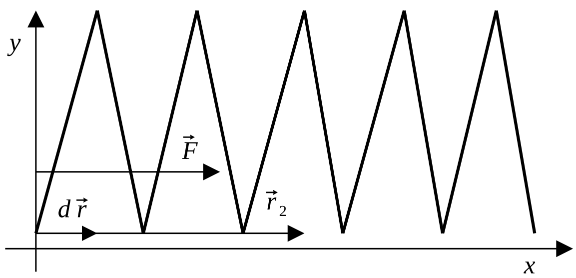

Fig. 5

The magnitude of the force exerted on a spring by an external force is directly proportional to the compression of the spring, it means

$$F = kx\ .$$

If a force of F_1 is required to compress a spring by x_1, the spring stiffness will be

$$F_1 = kx_1 \implies k = \frac{F_1}{x_1} \ .$$

Mechanical work expresses the path effect of a force

$$A = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \ ,$$

if the displacement vector and the force vector have the same direction, then

$$\vec{F} \cdot d\vec{r} = F dr \cos 0^\circ = F dr \ .$$

If the x axis has its origin at the point where the uncompressed spring is located and the direction of the x axis is the same as the direction of compression of the spring, then

$$dr = dx \ ,$$

$$r_1 = 0 \ ,$$

$$r_2 = x_0 \ .$$

The mechanical work will be

$$A = \int_{r_1}^{r_2} F\, dr = \int_0^{x_0} F\, dx = \int_0^{x_0} kx\, dx = \frac{kx_0^2}{2}$$

and after substituting the spring stiffness will be

$$A = \frac{F_1 x_0^2}{2x_1} \ .$$

After substituting numerical values

$$A = \frac{25\,000\,\mathrm{N} \ . \ (0{,}10\,\mathrm{m})^2}{2 \ . \ 0{,}01\,\mathrm{m}} = 12\,500\,\mathrm{J} = 12{,}5\,\mathrm{kJ} \ .$$

9. *A ball is suspended on a wire of length* $l = 0{,}5\,\mathrm{m}$. *What is the smallest horizontal velocity that must be given to it so that it can be deflected to its highest position while keeping the string taut?*

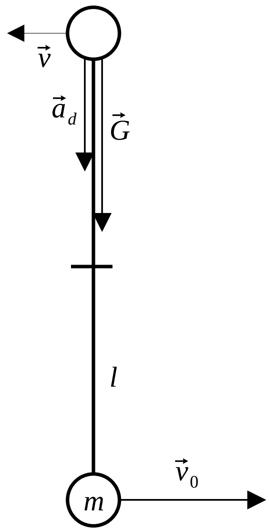

Fig. 6

If the ball at the highest position has a velocity of $\vec{v}$ and moves in a circle with radius l, it will have a centripetal acceleration of the magnitude

$$a = m\frac{v^2}{l} \ .$$

If it is also acted upon by a gravitational force

$$\vec{G} = m\vec{g} \ ,$$

then Newton's law of force will have the form

$$\vec{G} = m\vec{a} \ ,$$

in scalar form

$$mg = m\frac{v^2}{l} \ ,$$

from which it is possible to express the velocity of the ball at the highest position

$$v = \sqrt{gl} \ .$$

If the lowest position is a place with zero potential energy, then the total mechanical energy of the ball at the lowest position will be

$$E_{p0} + E_{k0} = \frac{mv_0^2}{2}$$

and at the highest position the total mechanical energy of the ball will be

$$E_p + E_k = mgh + \frac{mv^2}{2} \ ,$$

where the height of the ball will be

$$h = 2l \ .$$

From the law of conservation of mechanical energy, it follows

$$E_{p0} + E_{k0} = E_p + E_k \ ,$$

$$\frac{mv_0^2}{2} = 2mgl + \frac{mv^2}{2} \ ,$$

from which it is possible to calculate the velocity of the ball in the lowest position

$$v_0 = \sqrt{5gl} \ ,$$

after substituting numerical values

$$v_0 = \sqrt{5 \ . \ 9{,}81\,\mathrm{m\,s^{-2}} \ . \ 0{,}5\,\mathrm{m}} = 4{,}95\,\mathrm{m\,s^{-1}} \ .$$

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10. *Calculate the power of a car engine with a mass of* $m = 1200\,\mathrm{kg}$ *when the car is moving at a constant speed of* $v = 50\,\mathrm{km\,h^{-1}}$ *on a road with a five percent gradient.*

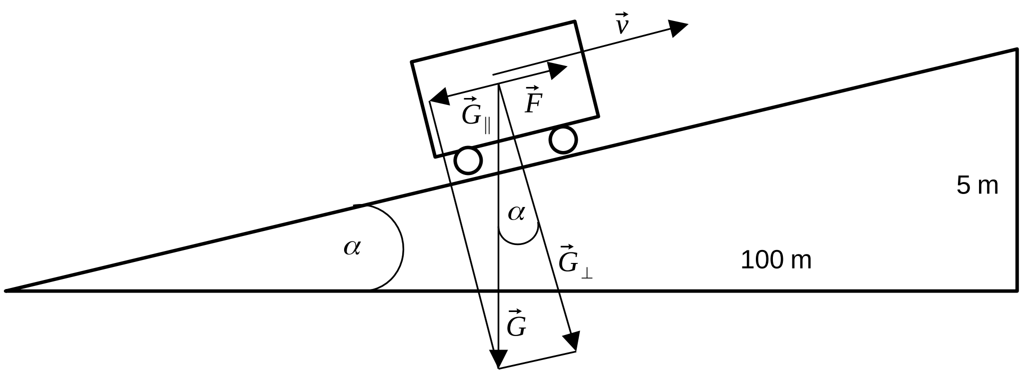

Fig. 7

The speed of the car is

$$v = 50\,\mathrm{km\,h^{-1}} = 13{,}9\,\mathrm{m\,s^{-1}}\ .$$

A five percent road gradient means that over a distance of $100\,\mathrm{m}$ the road will rise by $5\,\mathrm{m}$, therefore (Fig. 8)

$$\tan\alpha = \frac{5\,\mathrm{m}}{100\,\mathrm{m}} = 0{,}05$$

and the angle of a road inclination will be

$$\alpha = \arctan 0{,}05 = 2{,}86^\circ\ .$$

The engine power expresses the rate of work

$$P = \frac{\mathrm{d}A}{\mathrm{d}t}\ ,$$

if the force is constant, the engine power can be calculated as

$$P = \frac{\mathrm{d}(\vec{F}\cdot\vec{s})}{\mathrm{d}t} = \vec{F}\cdot\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} = \vec{F}\cdot\vec{v}\ .$$

The force of engine and the speed of car have the same direction, so

$$P = Fv\cos 0^\circ = Fv\ .$$

If the car is moving uniformly, the resulting force acting on the car must be zero, so the force of engine must be equal to the component of gravity that is parallel to the road

$$F = G_{\parallel} = mg\sin\alpha\ .$$

The power of engine will therefore be

$$P = Fv = mgv\sin\alpha\ ,$$

after substituting numerical values

$$P = 1200\,\mathrm{kg}\ .\ 9{,}81\,\mathrm{m\,s^{-2}}\ .\ 13{,}9\,\mathrm{m\,s^{-1}}\ .\ \sin 2{,}86^\circ = 8164\,\mathrm{W}\ .$$