29. MEASUREMENT OF THE MOMENT OF INERTIA BY THE TRIFILAR PENDULUM.

ASSIGNMENT

1. Measure the period of the trifilar pendulum
2. Using the formula (4.14) calculate the moment of inertia of the unknown body
3. Determine the moment of inertia of the basic disc

THEORETICAL PART

The moment of inertia is defined by

$$J = \int r^2 dm$$

where: *r* is the perpendicular distance from the axis of rotation to the element *dm*
dm is element of the mass of rigid body

The moments of inertia of rigid bodies with simple geometry are relatively easy to calculate provided the reference axis coincides with the axis of symmetry. The calculation of moments of inertia about an arbitrary axis can be somewhat cumbersome. The moment of inertia such a bodies can be obtain through a measurement of the period of a trifilar pendulum. The trifilar pendulum consist of a rigid body (disc) of mass *m* suspended from a three symmetric wires of length *l* attached at the top to a fixed support as is shown in Fig. 4.1.

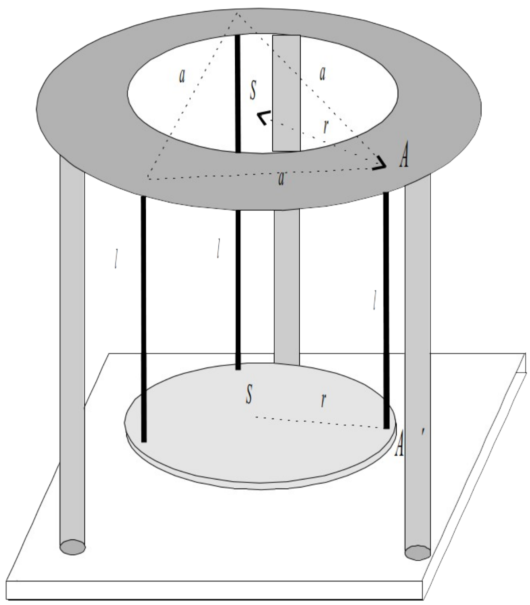

Fig. 4.1

The distances between the wires are *a*. The motion such a pendulum is given by

$$M_v = J\varepsilon \qquad (4.1)$$

where: M_v is the torque of the force to the axis of rotation, define by $M_v = r \times \vec{F}$

J is moment of inertia about an axis of rotation

ε is the angular acceleration given by $\vec{\varepsilon} = \dfrac{d^2\Phi}{dt^2}$

We shall examine some of properties of the trifilar pendulum. First we must determine the torque of the force to the axis of rotation. Situation of one wire is shown in Fig. 4.2

The forces acting on the mass of pendulum are the tension, T, acting along the string and the weight , mg . The weight vector is resolved into a component of weight tangent to the circle , $mg\sin\phi$, and the radial component (perpendicular to tangent component) ,

$mg\cos\Phi$. Note that the radial component is equal to the tension but they directions are opposite. The tangential component of the weigh always act to $\Phi = 0$, opposite the displacement. From the similar triangels of the Fig. 4.2 we have

$$\frac{F_t'}{\frac{1}{3}mg} = \frac{s}{l}$$

or

$$F_t' = \frac{mg}{3l}s \qquad (4.2)$$

When the system is displaced through the angle Φ we have

$$s = r\Phi$$

Than eq. (4.2) can be expressed as follows

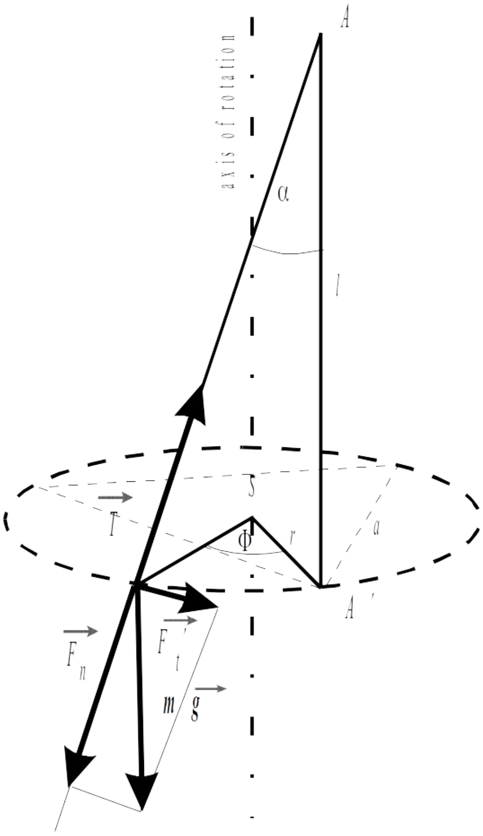

Fig. 4.2

$$F_t' = \frac{mg}{3l}r\Phi$$

The resultant force acting on the pendulum is

$$F_t = \sum_{i=1}^{N} F_{ti}' = \frac{mg}{l}r\Phi$$

and torque of this force to the axis of rotation is

$$M_v = F_t r = \frac{mg}{l}r\Phi r = \frac{mgr^2}{l}\Phi \quad (4.3)$$

Substituting eq. (4.3) into eq. (4.1) gives

$$-\frac{mgr^2}{l}\Phi = J\frac{d\Phi^2}{dt^2}$$

The mines sign indicates that the force act to the equilibrium position. Using the note

$$\frac{mgr^2}{Jl} = \omega^2 \qquad (4.4)$$

where: ω is the angular frequency

we obtain

$$\omega^2 \Phi + \frac{d^2\Phi}{dt^2} = 0 \qquad (4.5)$$

This is the second-order differential equation of the simple harmonic motion. Therefore, the solution can be written as

$$\Phi = A\cos(\omega t + \alpha) \qquad (4.6)$$

where: A is the maximum angular displacement

α is the phase constant

From Fig. 4.2 we can see that

$$r = \frac{a}{\sqrt{3}} \qquad (4.7)$$

The period of the harmonic motion is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3lJ}{mga^2}} \qquad (4.8)$$

Using eqs. (4.7) , (4.8) we can determine the moment of inertia of rigid body as

$$J = kmT^2 \qquad (4.9)$$

where: k is the constant characteristic for the using device. It dependens on the geometry of the pendulum according to the formula

$$k = \frac{ga^2}{12\pi^2 l} \qquad (4.10)$$

THE METHOD - PRACTICAL PART

Using the experimental arrangement of the trifilar pendulum we can have a method to measurement the moment of inertia of the various bodies. Let investigate the moment of inertia of the unknown body of mass m_1 . Let be the arbitrary shape of this body situated on the basic disc of mass m_0 and the radius R_0. The moment of inertia of the basic disc is related to the period of its oscillation , T_0 , as

$$J_0 = km_0 T_0^2 \qquad (4.11)$$

where: k is the constant given by eq. (4.10)

Now, we add to the basic disc an investigated body having a moment of inertia J_1 to the same axis of rotation. Then the resultant moment of inertia is given by

$$J = J_0 + J_1 \qquad (4.12)$$

If the period of oscillations of that system is T we have

$$J = k(m_0 + m_1)T^2 \qquad (4.13)$$

Then we obtain the formula for determination the moment of inertia of investigated body to the axis of rotation, J_1 , as

$$J_1 = k(m_0 + m_{1)})T^2 - km_0 T_0^2 \qquad (4.14)$$

MEASUREMENT

APPARATUS: measured body, trifilar pendulum, stopwatch, balance, meterstick

Measure the mass m_0 and the radius R_0 of the basic disc of trifilar pendulum. Measure the length of wire. Measure the distance a of the wires . Displaced the disc to one side through the angle of not more than 5^0 . Measure the period T_0 of the harmonic motion by the stopwatch a few complete oscillations. Determine the period T_0 . Put the unknown body on the basic disc of the trifilar pendulum and measure the period T of that system.

CALCULATION

Calculate the constant , k , from eq. (4.10) and the moment of inertia , J_1 , of the unknown body from eq. (4.14). Note the acceleration of gravity in Bratislava has a value $g = 9{,}806$ m.s^{-2} .

Calculate the moment of inertia of the basic disc , J_0 , by the formula

$$J_0 = \frac{1}{2} m_0 R_0^2 \qquad (4.15)$$

and compare this value with the experimental value given by formula (4.11).

3. Calculate the percent errors of your final results from degree of accuracy of the equipments which have been used.