26. Diffraction grating measurement of the wavelength of light.

Assignment

1. Measure the grating spacing.
2. Measure the wavelength of the sodium discharged tube.
3. Analyze the source of error in your result.

Theoretical part

Sources of visible light often produce many different wavelengths or colors. Light from sources utilizing hot solid metals filaments contain essentially a continuous distribution of wavelengths forming a white light. Light produced by a discharge in a gas of a single chemical element (Na, Hg, He and so on) contains only a limited number of discrete wavelength components, which are characteristic of the element. There are several methods, which can be used to separate a light source into its component wavelengths.

A very useful device for analyzing light source is the diffraction grating. A transition diffraction grating is a piece of transparent material on which has been ruled a large number of equally parallel lines. The distance between lines is called *the grating spacing* d , and it is usually only a few times as large as a typical wavelength of visible light. Wavelengths of visible light are in range from about 4×10^{-7} to 7×10^{-7} meters. It is customary to express the wavelength of light in units of *nanometers*, where $1\,\mathrm{nm} = 10^{-9}\,\mathrm{m}$. Starting from short wavelength and going to long wavelength the order of colors is violet, blue, green, yellow, orange and red.

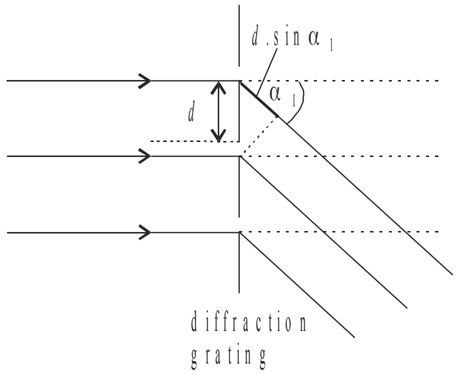

Fig. 8.1

Light rays which strike the transparent portion of the grating between the ruled lines will pass through the grating at all angles with respect to their original path. *If the deviated rays from adjacent ruling on the grating are in phase, an image of the source will be performed.* This will be true when the adjacent rays differ in path length by a integral number of wavelength, λ, there will exist a series of angles at which an image is formed. The first time an image is formed when *the path difference* between adjacent ruled lines is equal to wavelength λ. From Fig.8.1 follows that condition will be true at an angle α_1 such that the equation

$$\lambda = d \sin \alpha_1 \tag{8.1}$$

is satisfied. At some larger angle α_2, when the path difference adjacent rays from adjacent ruled lines is exactly equal 2λ, then the equation

$$2\lambda = d \sin \alpha_2 \tag{8.2}$$

is satisfied. In general, an image will be performed at any angle α_m for which adjacent rulings have a path difference equal to $m\lambda$ and it is describe by the equation

$$m\lambda = d \sin \alpha_m , \tag{8.3}$$

where m is an integer called *the order number*. Eq. (8.3) expresses the condition for maximum of the interference pattern at the angle α_m.

If we use a collimator placed between a grating and the screen the interference maximum is found on the screen. The intensity of these maximum (bands) depends on the order of maximum. If the order of maximum increasing, the intensity decreases and vice versa.

If the incident light contains a several wavelength the m-th order maximum for each wavelength occurs a specific angle. All wavelengths are seen at $\alpha = 0$ corresponding to $m = 1$, is observed at angle that satisfied the relationship

$$\sin\alpha_1 = \frac{\lambda}{d}$$

and so on. The zeroth, first and second order maximum for monochromatic light is shown in Fig. 8.2. Note that if the source of light contains various wavelengths, a spectrum of lines will be observed at different portions for each order number.

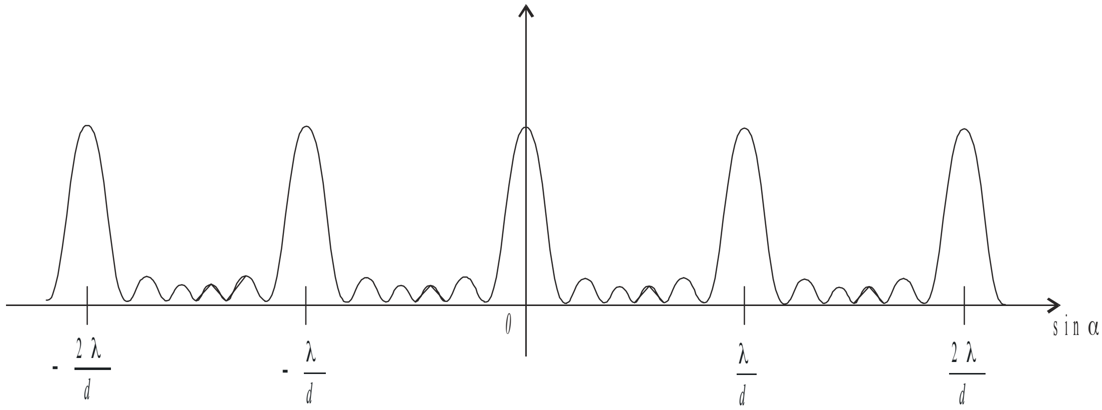

Fig. 8.2

The method-practical part

A simple experimental arrangement that is used to measure various orders of the diffraction pattern is shown in Fig. 8.3.

The light passes through the slit and parallel beam of the light from the collimator, which is perpendicular to the grating, is incident on the grating. The diffracted light leaves grating at angles that satisfied equation (8.3) The telescope is used to measure the maximum for the various orders.

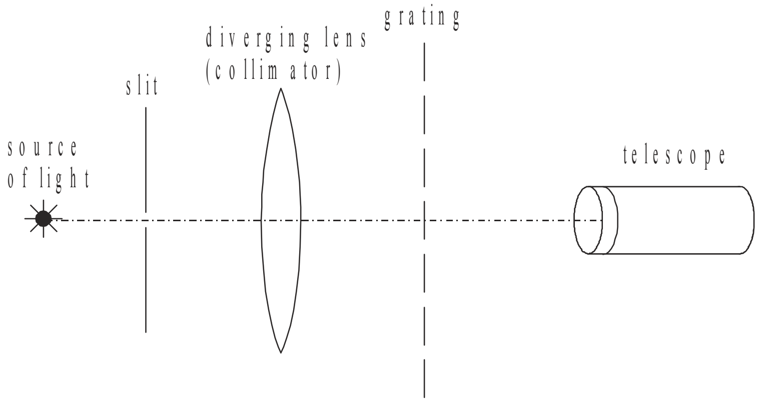

Fig. 8.3

If the focal length of lens of objective of telescope is f and s_m is the position of the m order maximum to the zeroth order maximum then from Fig. 8.4 follows

$$\sin\alpha_m = \tan\alpha_m = \frac{s_m}{f} \qquad (8.4)$$

From this equation

$$s_m = f\sin\alpha_m \qquad (8.5)$$

or

$$\sin\alpha_m = \frac{m\lambda}{d} \qquad (8.6)$$

Inserting eq. (8.5) into eq. (8.3) gives very important relation between the wavelength and position of the pattern

$$s_m = \lambda\frac{mf}{d}\,. \qquad (8.7)$$

Then the distance between adjacent maximum s_m and s_{m-1} is

$$s_m - s_{m-1} = \frac{\lambda f}{d}\left[m - (m-1)\right] = \lambda\frac{f}{d} \qquad (8.8)$$

or

$$s = \lambda\frac{f}{d}, \qquad (8.9)$$

where $s = s_m - s_{m-1}$ is distance between adjacent maximum finding on the scale of telescope, λ is the wavelength of the monochromatic light, f is the focal length of the collimator of telescope and d is *a split spacing of the grating*. This constant is defined to be as the inverse of the number of lines per centimeter, i. e.

$$d = \frac{1}{N}\,\text{cm}\,.$$

For example, if the grating contains 5000 lines per centimeter its split spacing has a value

$$d = \frac{1}{5000} = 2 \times 10^{-4}\ \text{cm}\,.$$

Measurement

Apparatus: sodium lamp, optical bench, slit, diffraction grating, telescope, microscope.

Experimental procedure: Set up the telescope to infinity. Set up the experimental device using the scheme given by Fig 8.4 so that the image of the slit must be sharp in telescope. Put a diffraction grating in front of objective of telescope. It may be necessary to rotate the grating in its holder to place the image in horizontal. Record the position of several lines on the right and on the left with respect to the zeroth order maximum. Measure the split spacing of grating (grating spacing) d a few times. Read the measured values into data tables. Focal length of the lens of objective is introduced on the border of objective.

Calculation: Calculate the mean value of the grating spacing d_{av}. Calculate the average distance from the zeroth order maximum to each image on the left s_l and on the right s_r as $\bar{s} = \frac{s_r + s_l}{2}$. Then calculate the wavelength of the sodium light λ_{Na} from eq. (8.9). The table value of the yellow spectral line of sodium is $\lambda_{\text{Na}} = 589.3\ \text{nm}$. Analyze the source of errors in your experiment.

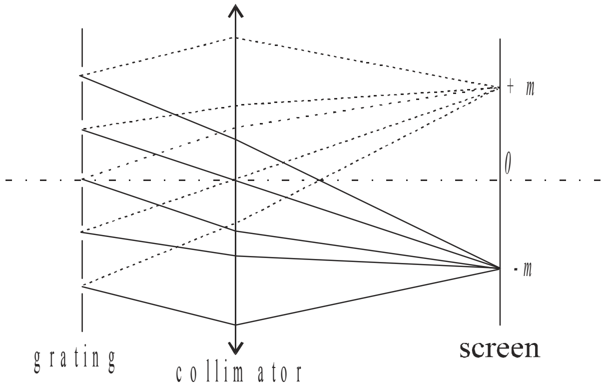

Fig. 8.4