## 25. Measurement of the wavelength of light by Newton`s rings.

## Assignment

1. Measure the radius of curvature of plane-convex lens.
2. Measure the wavelength of the monochromatic light.
3. Analyze the source of errors.

## Theoretical part

Visible light, the most familiar form of electromagnetic waves, may be defined as the part of spectrum that human eye can detect. The various wavelength of visible light are classified with color ranging from violet $\left(\lambda \sim 4 \times 10^{-7} \mathrm{~m}\right)$ to red ( $\lambda \sim$ $7 \times 10^{-7} \mathrm{~m}$ ).

Today, scientists view light as having a dual structure. Sometimes light behaves as if it were particle-like (for example photoelectric effect), and sometimes it behaves if it were wave-like (diffraction, polarization or interference effects).

We shall study the phenomena of interference as they apply to the light. Interference of light waves is the result of the linear superposition of two or more waves at a given point. Two waves could add together constructively or destructively. In constructive interference, the amplitude of the resultant wave is greater than that of either of the individual waves. In destructive interference the resultant amplitude is less than of either if the individual waves. In order to observe sustained interference in light waves, the following conditions should be met:

1) The sources of the light must be coherent, that is, they must maintain a constant phase with respect to each other.
2) The sources of light must be monochromatic, that is, of a single wavelength.
3) The principle of superposition must be applied.

The interference of light waves was at first time demonstrated by Thomas Young in 1801. This experiment is called Young s double split experiment.

Suppose that there are two harmonic waves in form

$$
\begin{align*}
& y_{1}=A_{1} \sin \left(\omega t+k r_{1}\right) \\
& y_{2}=A_{2} \sin \left(\omega t+k r_{2}\right) \tag{7.1}
\end{align*}
$$

each with the same frequency and speed, coexisting in space, where $r_{1}$ and $r_{2}$ are the distances from the sources of the two monochromatic waves to the point P of observation as is shown in Fig. 7.1. The symbol in the equation (7.1) is called the wave number defined by relation

$$
\begin{equation*}
k=\frac{2 \pi}{\lambda} \tag{7.2}
\end{equation*}
$$

where $\lambda$ is the wavelength of light.


Fig. 7.1
The resultant disturbance in the linear superposition of these waves is

$$
\begin{equation*}
y=y_{1}+y_{2}=A_{1} \sin \left(\omega t+k r_{1}\right)+A_{2} \sin \left(\omega t+k r_{2}\right) \tag{7.3}
\end{equation*}
$$

Rearranging eq.(7.3) and comparing with the general form of equation for the harmonic wave gives the relation between resultant amplitude $A$ and amplitudes of both interfering waves $\boldsymbol{A}_{1}$ and $\boldsymbol{A}_{2}$ as

$$
\begin{equation*}
A^{2}=A_{1}^{2}+A_{2}^{2}-2 A_{1} A_{2} \cos \left(r_{2}-r_{1}\right) \tag{7.4}
\end{equation*}
$$

An additional term $2 A_{1} A_{2} \cos \left(r_{2}-r_{1}\right)$ is known as interference term. From eq. (7.4) we can see that $A$ falls between values of $A_{1}+A_{2}$ and $A_{1}-A_{2}$ depending on the value of $k\left(r_{2}-r_{1}\right)$. This value may be

$$
\begin{equation*}
k\left(r_{2}-r_{1}\right)=2 m \pi \tag{7.5}
\end{equation*}
$$

or

$$
\begin{equation*}
k\left(r_{2}-r_{1}\right)=(2 m+1)_{\pi} \tag{7.6}
\end{equation*}
$$

where $m$ is either a positive or a negative integer, i. e. $m= \pm 1, \pm 2, \pm 3, \ldots$. This integer is called the order number, and $r_{2}-r_{1}=\Delta$ is called the optical path difference (see Fig. 7.1).

In the first case (eq.(7.5)) we have maximum reinforcement of the two wave motions, or constructive interference (bright fringes), and the second case (eq.(7.6)) minimum alternation, destructive interference (dark fringes). If we insert the value of wave number given by eq.(7.2) into eq.(7.6) we have condition for dark fringes in form

$$
\begin{equation*}
\Delta=(2 m+1) \frac{\lambda}{2} \tag{7.7}
\end{equation*}
$$

If $m= \pm 1$ the maximum is called zeroth-order maximum of destructive interference. If $m= \pm 2$ the maximum is called second-order maximum of interference, and so forth.

## The method-practical part

We shall measure the wavelength of monochromatic light by the optical device calling Newton`s glasses. Scheme of the Newton`s glasses is shown in Fig. 7.2.

In this arrangement, the air film between the glass surfaces varies by thickness of air layer from zero at the point of a contact P to some value $h_{\max }$ at point Q . If the radius $R$ of the curvature is very large comparing with $h$ and if the system is viewed from above using light of wavelength $\lambda$, a pattern of light and the dark rings are observed. This circular fringes, discovered by Newton, are called Newton`s rings. This interference effect is due to the combination of ray reflected from the plane glass plate and the ray reflected from the curved part of the plane-convex lens as is shown in Fig. 7.3.


Fig.7.2


Fig.7.3

From the detail figure in Fig. 7.3 we see that first ray undergoes a phase change of $180^{\circ}$ upon reflection, since it is reflected from a medium of higher index of refraction, whereas second ray undergoes no phase change.

Let us apply the rule given by eq. (7.7) to the air film. Assume that we need examine the two reflected beams 1 and $1^{\prime}$. Since the ray reflected on the bottom surface is reflected by a medium whose index of refraction is greater than that of the air gap, the ray changes phase by $180^{\circ}$. So the path difference between two reflected rays will be given

$$
\begin{equation*}
\Delta=2 h+\frac{\lambda}{2} \tag{7.8}
\end{equation*}
$$

where $h$ is the high of air film and $\frac{\lambda}{2}$ is a phase change upon reflection from a medium of higher index of refraction (glass plane). Note that eq. (7.8) is exact for $h$ $\ll r$ and $r \ll R$, where $R$ is the radius of the plane-convex lens and $r$ is the radius of the interference circle.

Now we would find the relationship between $r$ and $h$. Using the geometry in Fig 7.2 one can obtain expression for the radius of rings $r$ in form

$$
\begin{equation*}
r^{2}=R^{2}-(R-h)^{2} \tag{7.9}
\end{equation*}
$$

If $h \ll R$ the relation (7.9) transforms into form

$$
\begin{equation*}
r=\sqrt{2 R h} \tag{7.10}
\end{equation*}
$$

When we combine eqs (7.7), (7.8) and (7.10) we give the expression for radii of dark rings as

$$
\begin{equation*}
r_{m}=\sqrt{m R \lambda} \tag{7.11}
\end{equation*}
$$

where $m$ is the order number. From this equation follows the method of measurement of the wavelength $\lambda$ of monochromatic light if the radius of the plane-convex lens is known.
Remark: The circular pattern is obtained only when the lens is ground to a perfectly symmetric curvature. The digression from such symmetry gives an indication of how the lens has the imperfections.


Fig. 7.4

## Measurement

Apparatus: Newton`s glasses, plane glass, sodium lamp, mercury lamp, filters, microscope with device to record the position of the interference rings.

Experimental procedure: The complete experimental device for measurement of the wavelength of monochromatic light is shown in Fig. 7.4. An arrangement for producing an interference pattern contains a single light source. The beam of light is focused by the lens. The monochromatic source is splitted into rays by abeam splitter at angle of $45^{\circ}$ relative to the incident light beam. The beams fall on the Newton's glasses and we can observe the interference pattern by the microscope, as is shown in Fig. 7.4. Use the sodium lamp ( $\lambda_{\mathrm{Na}}=589.3 \mathrm{~nm}$ ) to measure the radius of curvature $R$ of Newton`s plane-convex lens. Focus the line in ocular of microscope and adjust the objective of microscope till the Newton`s ring are focused. Measure the positions of dark rings to the right $d_{r}$ and to the left $d_{l}$ from the zeroth-order maximum a few times. Read the measured values into data table Tab.1.1.

| $m$ | $d_{r}$ | $d_{l}$ | $d_{m}=d_{r}-d_{l}$ | $r_{m}=d_{m} / 2$ | $r_{m}^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |

Tab.7.1
Calculation: Calculate the radii of $r_{m}$ like difference between position of dark ring on the left and right side. Calculate the radius $R$ the Newton`s lens using eq. (7.11). Rearranging this equation gives

$$
R=\frac{r_{m}^{2}}{m \lambda}
$$

or

$$
\begin{equation*}
r_{m}^{2}=R m \lambda \tag{7.12}
\end{equation*}
$$

Because this equation is equation of linear we can determine the value of $R$ by linear regression. The slope of $r_{m}^{2}$ versus $m$ graph determines the value $\lambda$ as is shown in Fig. 7.5. Repeat this procedure using the unknown source of light (Hg lamp by a filter). From the slope of linear that equals $\lambda R$ we can calculate the wavelength of $\lambda_{\mathrm{Hg}}$ using the value of $R$ from previous measurement. Analyze the errors in your


Fig.7.5
measurements.

