24. Measurement of a focal length of lens. Investigation of chromatic and spherical aberration of lens.

Assignment

1. Measure the focal length of the converging lens.
2. Measure the focal length of the diverging lens.
3. Investigate the chromatic and spherical aberration of lens.
4. Analyze the source of errors in your experiment.

Theoretical part

Telescopes, microscopes, cameras, projectors etc. are just the instrument, which depend on the image-forming abilities of lens.

A lens is a transparent medium so shaped that it can produce an image by refracting light that comes from object. A lens is bounded by two curved usually spherical surfaces, although one of the faces of the lens may be plane. The image may be real or virtual, erect or inverted or smaller or the same size as the object. Lenses are divided into two groups.

1. *A converging lens* having a positive focal length and its acts to bring incoming parallel beam of light to a single focal length, as is shown in Fig. 6.1.
2. *A diverging lens* having a negative focal length. This lens has curved surfaces so shaped that the diverging rays appear to originate from a single point, *the virtual point F* (Fig. 6.2).

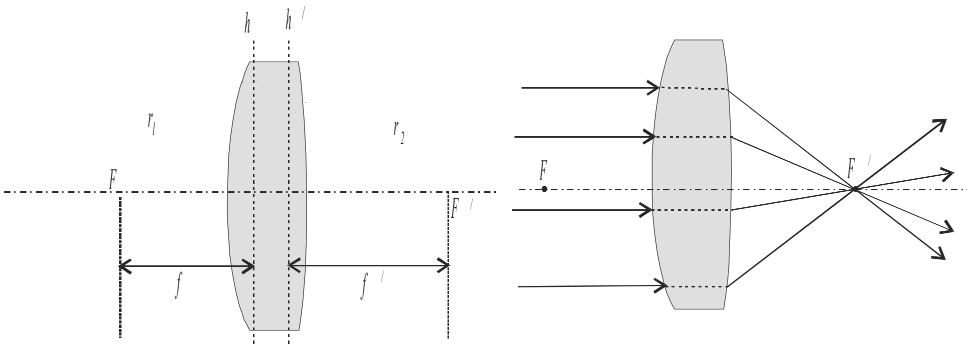

Fig. 6.1

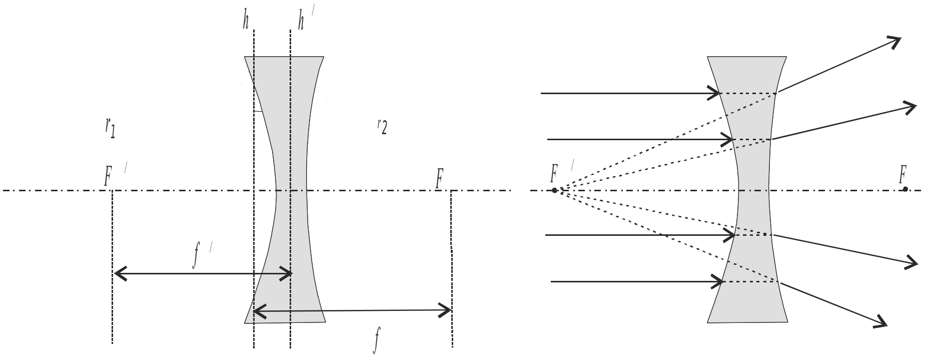

Fig. 6.2

A lens is also classified as to whether *it is thin or thick*, that is, whether or not is thickness effectively negligible.

For the most part, all surfaces are rotationally symmetric about a common axis, which is called *the optical axis*. The focal length of a particular lens depends both on the index of refraction n of its material relative to that of medium it is in and on the radii of curvature r_1 and r_2 of its surfaces. For thin lens the equation for determination the focal length is in form

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right), \tag{6.1}$$

where $n = \frac{c}{v}$ is the index of refraction, c is the velocity of the light in free space, v is the velocity of the light in the medium (lens). Equation (6.1) is called *the Lensmaker`s equation* for a thin lens.

Sign conventions for this formula are as follows:

1. if the curvature of lens is curved outward (convex) then the radius is positive and lens is converging
2. if the curvature is curved inward (concave) then the radius is negative and lens is diverging.

From this follows, that a lens may have *positive or negative focal length*, depending on its shape. A positive focal length signifies a converging lens and a negative one signifies a diverging lens. A lens has two focal points, one of each side the distance *f* from its center. The focal point, F, on the side of the lens from which the light comes is *the near focal point* or *object focal point* and one on the other side of the lens, F', is *the far focal point* or *image focal point*.

From the process of image formation, the distance of the image from the lens is called *the image distance*, s'. The distance of the object from the lens is called *the object distance*, S . Situation is shown in Fig. 6.3.

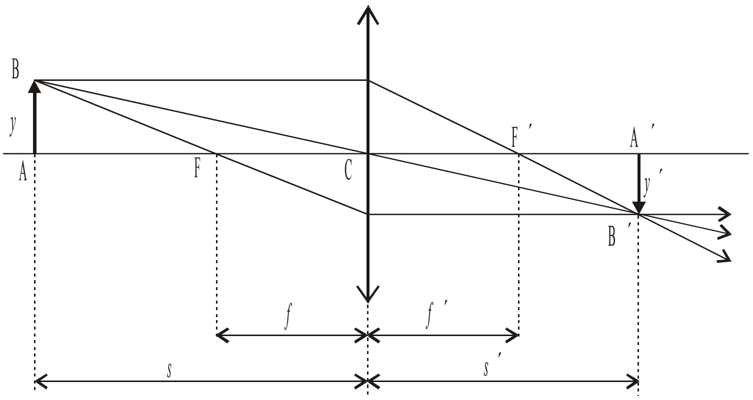

Fig. 6.3

The relationship between the object distance S and s', and focal length is given by the equation

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}. \qquad (6.2)$$

Note that this equation is valid both for converging ($f > 0$) and for diverging ($f < 0$) lens. Normally the object distance is considered positive. In that case a positive value for an image distance means that the image is on the opposite side of the lens from the object and image is real. A negative value for an image distance means that the image is on the same side as the object. The construction of the image for the various distances of an object from a lens is shown in Fig. 6.4 and Fig. 6.5. From the Fig. 6.5 follows that we must use the converging lens to formed the real object.

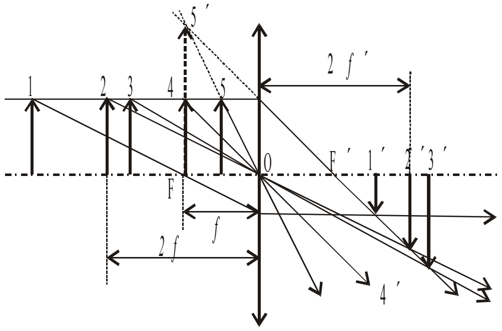

Fig. 6.4

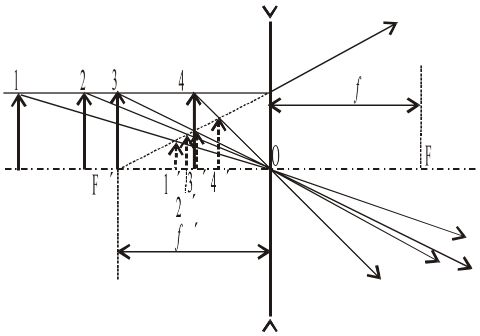

Fig. 6.5

If two lenses with focal length of f_1 and f_2 are placed in contact, the combination of the two in contact act as a single lens of *effective focal length* f_e. The effective focal length is related to the individual focal length of the lenses of f_1 and f_2 by equation

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2}. \tag{6.3}$$

Remember that this equation holds for any combination of converging and diverging lenses, respectively. If the individual lenses are converging, then the focal length f_e will be converging. If one of the lenses is converging and the other is diverging, the focal length f_e can be either converging or diverging depending upon the values f_1 and f_2. This fact can be used to determine the focal length of an unknown diverging lens if it is used in combination with a converging lens whose focal length is short enough to produce a converging combination as is shown in Fig. 6.3.

It is purposive to introduce linear magnification defining as

$$m = \frac{\text{image height}}{\text{object height}} = \frac{y'}{y}, \tag{6.4}$$

where y is the dimension of the object, y' is the dimension of the image. From the equation (6.4) follows that m is positive in the case of erect image and negative if the image is inverted.

Now we can find the relation between linear magnification m, the object distance and image distance. As is shown in Fig. 6.3 the triangle ABC is similar to the triangle $\mathrm{A'B'C}$. From it follows that

$$\frac{s'}{s} = \frac{y'}{y}, \tag{6.5}$$

where s' is the distance of image from the center of converging lens C, s is the distance of object from the center C.

Inserting expression (6.5) into equation (6.4) gives

$$m = -\frac{y'}{y} = -\frac{s'}{s}, \tag{6.6}$$

where we use the sign convention for converging lens.

Lens aberrations

The image formed by a lens is never an exact replica of its object. Of the variety of imperfections to which such an image is subject, the most familiar is *the spherical and chromatic aberrations.*

Spherical aberration is result from the fact that the focal points of light rays far from the optical axis of lens are different from the focal points of rays of the same wavelength passing near the center of the lens as is shown in Fig. 6.6.

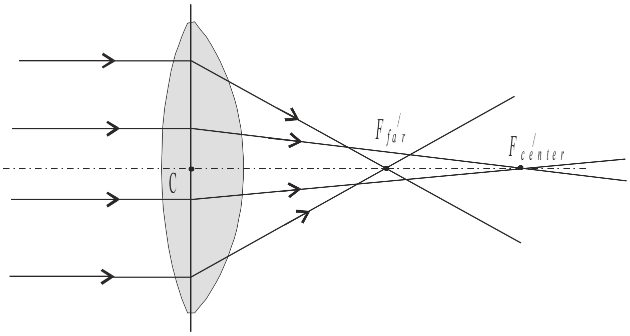

Fig. 6.6

Rays near the middle of the lens (point C) are imaged at greater distances from the lens that the rays at the edges. Hence, that is no single focal length for a lens. Sharper images are produced as the aperture size is reduced, since only the central portion of the lens is exposed to the incident light.

The fact that the wavelengths of light refracted by a lens focus at different points give rise *a chromatic aberration* as is shown in Fig. 6.7.

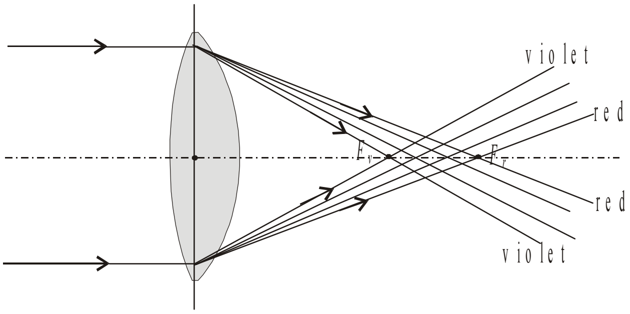

Fig. 6.7

When white light passes through the lens, one finds, that blue light rays are refracted more that red ones, for example. From this follows that index of refraction of thin lens depends on the wavelength. It can be reduced by using a combination of the converging and diverging lenses made from different type of glasses.

The method-practical part

A. Measurement of the focal length of a converging lens

The equation (6.2) gives the simple method of measurement the focal length of converging lens. Using the sign convection we get

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$$

or

$$f=\frac{ss'}{s+s'}. \qquad (6.7)$$

If we introduce the linear magnification m into this equation (see eq. 6.6) we have

$$f=\frac{1}{1-m}s', \qquad (6.8)$$

where $m=-\frac{s'}{s}$. Note that both these formulae are valid only if the radii of bounded spherical surfaces are the same, i. e. $r_1=r_2$.

Measurement

Apparatus: optical bench, source of light, converging lens, screen.

Experimental procedure: Place the lens in a lens holder, source of light, object and screen on the optical bench. Adjust the distance from the lens to the screen until a sharp real image of the distance object is formed on the screen. Measure image distance s' and object distance s and measure the magnitude of image y' and magnitude of an object y. Repeat the measurement for various distances between object and converging lens. Record your measurement in the data table Tab. 6.1.

Calculation: Using eqs. (6.7) and (6.8) calculate the focal length of the converging lens from the measured data. Calculate the average value of focal length f and from eq. (6.6) calculate the linear magnification m.

i	s	s'	y	y'	f	m

Tab.6.1

B. Measurement of focal length of the diverging lens

If we showed the image formation of a diverging lens is virtual. This virtual image can be transformed to the real image by the thin converging lens as is shown in Fig. 6.8.

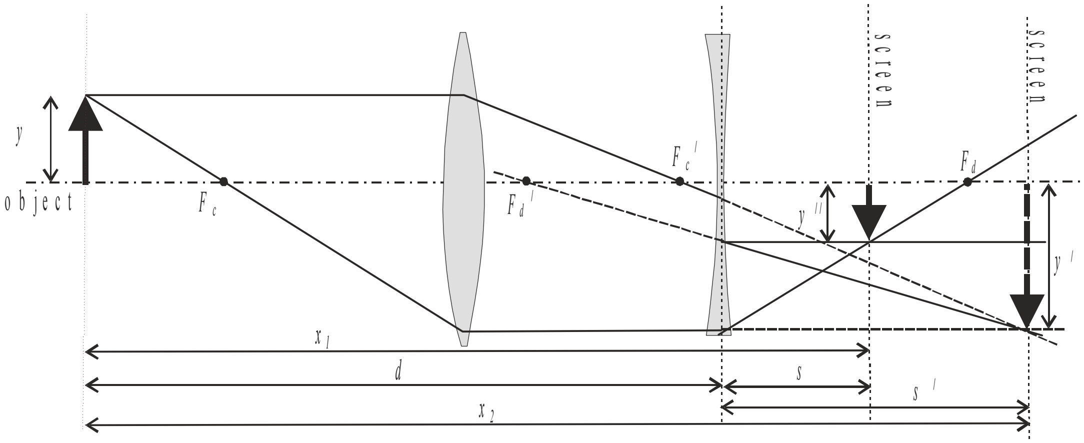

Fig. 6.8

If the diverging lens is placed between a converging lens and real image y' created by converging lens that the real image y' will be a virtual image for the diverging lens. Then the diverging lens produces an enlarged real image y'' of the object. Then the focal length of the diverging lens is given by the equation

$$f_{div} = -\frac{ss'}{s' - s}, \qquad (6.9)$$

where $s = x_1 - d$, $s' = x_2 - d$ and d is the distance between object and diverging lens.

Measurement

Apparatus: optical bench, source of light, diverging lens, converging lens and screen.

Experimental procedure: Place the converging lens on the optical bench at the distance larger than $2f$. Read the position of the screen x_1 if the sharp image on the screen is found. Place diverging lens between converging lens and screen and find the sharp image by the shift of the screen. Read the position of the diverging lens d and new position of the screen x_2 . Note that converging lens must be at rest. Repeat this measurement a few times. Record your measurement in the data table Tab. 6.2.

Calculation: Using eqs. (6.9) and (6.10) calculate the focal length of diverging lens and then average value of focal length of diverging lens.

i	d	x_1	x_2	s	s'	f

Tab.6.2

C. Investigation of the chromatic aberration of lens

This investigation requires a few filters with various wavelengths. The principle is based on the fact, that the light of various wavelengths is focused at a different place along the optical axis. If we can put, for example, the red filter in front of the object we can determine the focal length of the lens for red light finding the sharp image of the object and using the Lensmaker`s equation.

Measurement

Apparatus: optical bench, source of light, filters, converging lens, screen.

Experimental procedure: Place the red filter in front of converging lens on the optical bench and find the position of the screen if the sharp image is formed. Read the position of the image and the object from the center of converging lens. Repeat the measurement for blue and green filters.

Calculation: Draw the graph of focal point F versus wavelength λ of light used. Analyze the result of your experiment.

D. Study of the spherical aberration of lens

For the study of the spherical aberration we have to use, for example, the red filter to have a monochromatic light because it not to be confuse spherical aberration with chromatic aberration. If we cover the converging lens with the diaphragm, which has an aperture of about 1 cm at center we can determine the position of the sharp image on the screen. If we replace the central aperture with a central disc, which allows light to pass through the outer portion of the lens, we can predetermine the new image distance from the center of lens.

Measurement

Apparatus: optical bench, source of light, filters, diaphragm, converging lens, screen.

Experimental procedure: Use the red filter to measurement of the spherical aberration to exclude the chromatic aberration. Place the diaphragm with small aperture in front of the converging lens and find the sharp image on the screen. Read the position of the image.

Calculation: Analyze the spherical aberration of the converging lens.