23. Measurement of the wavelength and the velocity of the sound by the method of acoustic interferometer.

Assignment

1. Measure the wavelength of the sound.
2. Determine the speed of the sound in the air.
3. Discuss the source of errors.

Theoretical part

Sound waves are the most important example of longitudinal waves. They can travel through any material medium with a speed that depends on the properties of the medium. As sound waves travel through a medium, the particles in the medium vibrate to produce density and pressure changes along the direction of motion of the wave. The displacement that occurs as a result of sound waves involve the longitudinal displacement of individual molecules from their equilibrium positions. We studied these phenomena in exercise of 4 (Kundt's tube).

There are three categories of longitudinal mechanical waves that cover different ranges of frequency:

1. *Audible waves* are sound waves that lie within the range of sensitivity of the human ear (20 Hz to 20000 Hz). They can be generated in a variety of ways: musical instruments, loudspeaker, etc.

2. *Infrasonic waves* are moves with frequencies below the audible range. For example, earthquake waves.

3. *Ultrasonic waves* which have the frequencies above the audible range. They can be generated, for examples, they can be generated by inducing vibrations in a quartz crystal with an applied alternative electric field.

Sound waves are compressional wave travelling through a compressible medium. The wave speed depends on a classical properties of the medium (as we know from exercise 4) as

$$v = \sqrt{\frac{elastic\ \ property}{inertial\ property}}\ .$$

The method-practical part

One of the most methods of measurement of speed of the sound in air is the measurement by the acoustic interferometer. The acoustic interferometer is the device which measure the phase difference between two acoustic waves with the same frequencies and various phases. *The technique is based in comparison the frequency of a unknown source to that of standard oscillator* and which can be used to determine the relative phase as well makes use of are know as Lissajous figures.

Let us now assume two harmonic waves moving along two perpendicular directions given by equations

$$x = x_0 \cos \omega t$$
$$y = y_0 \cos(\omega t + \varphi), \qquad (5.1)$$

where: x_0, y_0 are the amplitudes of the waves, ω is the anglular velocity, t is the time, φ is the phase difference between these longitudinal waves.

Rewriting eqs.(5.1) gives

$$\frac{y}{y_0} = \frac{x}{x_0} \cos \varphi - \sin \omega t \sin \varphi$$

or

$$\frac{y}{y_0} - \frac{x}{x_0} \cos \varphi = - \sin \omega t \sin \varphi\ . \qquad (5.2)$$

If we square this equation we get

$$\left(\frac{y}{y_0} - \frac{x}{x_0} \cos \varphi \right)^2 = (- \sin \omega t \sin \varphi)^2\ . \qquad (5.3)$$

Using the trigonometric identity we give

$$\sin^2 \omega t = 1 - \cos^2 \omega t = 1 - \left(\frac{x}{x_0}\right)^2 \tag{5.4}$$

since $\cos \omega t = \frac{x}{x_0}$. Inserting eq.(5.4) into eq.(5.3) gives

$$\left(\frac{y}{y_0}\right)^2 + \left(\frac{x}{x_0}\right)^2 - \frac{2xy}{y_0 x_0}\cos\varphi = \sin^2\varphi \tag{5.5}$$

Eq.(5.5) is the equation of the ellipse. *It means that the interference of two oscillators perpendicular to each other which have the same frequencies* ω *causes that any point will be moved along the ellipse. The shape and place of this curve depends on the amplitudes* x_0, y_o *and on the phase difference between them*. Fig.5.1 shows graphically how the ellipse is formed in the case where the phase difference is 45° .

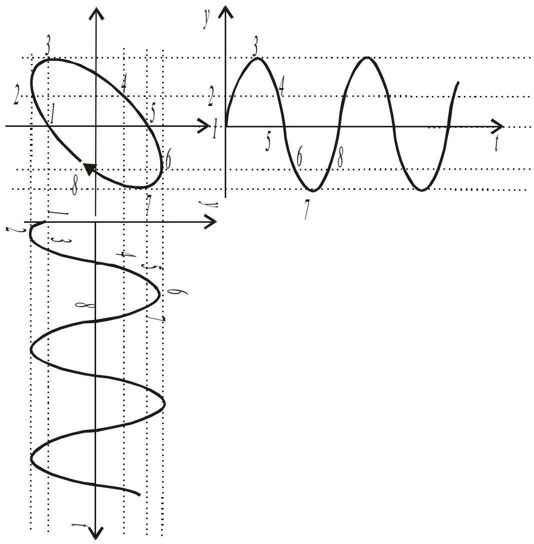

Fig.5.1

Using the same approach we can obtain the Lissajous figures for phase difference of 0°, 90°, 135° and 180° as is shown in Fig.5.2. The

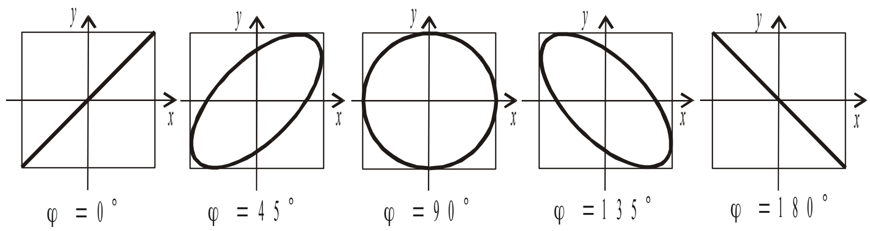

Fig.5.2

scheme of the acoustic interferometer is shown in Fig.5.3. The acoustic interferometer consists from the tonal generator (*TG*), oscilloscope (*OSC*), reproducer (*R*), microphone (*MP*) and optical bench.

Oscillations spreading from the loudspeaker are intercepted by the microphone where

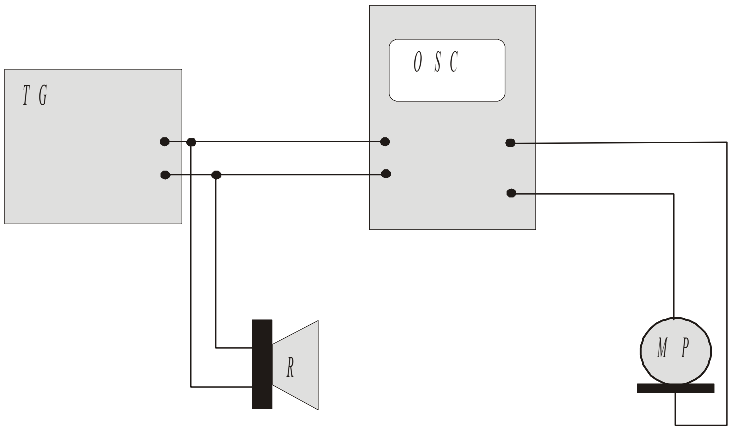

Fig.5.3

are transformed into electric oscillations. These are driven by one sinusoidal channel of oscilloscope. The frequency of standard oscillator is driven by the second channel of the oscilloscope. The oscillations at the two different distances between the microphone and amplifier may be described by equations

$$x_1 = x_{01}\cos(\omega t - \varphi_1) = x_{01}\cos\omega\left(t - \frac{d_1}{v}\right)$$
$$x_2 = x_{02}\cos(\omega t - \varphi_2) = x_{02}\cos\omega\left(t - \frac{d_2}{v}\right), \qquad (5.6)$$

where v is the velocity of the sound in air, d_1, d_2 are the two various distances between the microphone and amplifier, x_{01}, x_{02} are the amplitudes of oscillations at the distance d_1, d_2, separately. From eqs.(5.6) the phase difference is equal

$$\Delta\varphi = \frac{\omega}{v}(d_1 - d_2) \tag{5.7}$$

If we assume that the path difference between these two oscillations is equal to the wavelength λ, i.e.

$$d_1 - d_2 = \lambda \tag{5.8}$$

we find that the phase difference is reduced to

$$\Delta\varphi = 2\pi \tag{5.9}$$

Inserting eqs.(5.8), (5.9) into eq.(5.7) gives

$$v = \lambda f \tag{5.10}$$

where $f = \frac{\omega}{2\pi}$ is the frequency of the sound wave. From eq.(5.10) results the method of measurement of the speed of sound in air.

Measurement

Apparatus: tonal generator, microphone, reproducer, oscilloscope, optical bench.

Experimental procedure: Connect the apparatus as is shown in Fig.5.3. Measure two positions of the microphone in which is the phase difference of oscillation $\Delta\varphi$ equal 2π. Repeat the measurement for the various frequencies and record them into data table Tab.5.1. Measure the temperature of the air in laboratory.

Calculation: From the data table calculate the velocity of the sound in the air using eqs.(5.8), (5.10). Compare your result with the value of the speed of sound in the air from the table recording the speed of sound in the air versus the temperature. Analyze the source of errors of this experiment.

i	f	d_1	d_2	λ	v

Tab.5.1