22. Measurement of the velocity of sound by resonance method. Measurement of modulus elasticity in tension by the method of Kundt`s tube.

Assignment

1. Measure the velocity of the sound in air.
2. Measure the velocity of the sound in metal rod.
3. Determine the Poisson`s constant in air.
4. Determine the modulus of elasticity of the rod.
5. Give complete analysis of the errors in your experiment.

Theoretical part

A classical travelling wave is a self-sustaining disturbance of a medium, which moves through the space transporting energy and momentum. When we look closely at real waves (such a waves on string), we see composite phenomena comprised of vast numbers of particles moving in concert. The media supporting these waves is atomic and so the waves are not continues entities in and of themselves. The most familiar waves, and the easiest to visualize are the mechanical waves, among which are waves on string, surface waves on liquids, sound waves in air and compression waves in both solid and fluids. Sound waves are longitudinal - the medium is displaced in the direction of motion of the wave. Waves on the string and electromagnetic waves are transverse - the medium is displaced in a direction perpendicular to that of the motion of the wave. In all cases, although the energy-carrying disturbance-advances through the medium, the individual participating atoms remain in the vicinity of their equilibrium position.

We shall take an interest in sound waves. These ones are longitudinal harmonic waves. These may be described by the equation

$$y = y_0 \sin(kx \pm \omega t), \tag{4.1}$$

where k is the wave number defined as

$$k = \frac{2\pi}{\lambda} \tag{4.2}$$

and

$$\omega = kv = \frac{2\pi}{\lambda} v \tag{4.3}$$

is *the angular frequency of the wave*. Because $\omega = 2\pi f$, where f is frequency with which the physical situation varies at every point x, we have the important relation

$$\lambda f = v \tag{4.4}$$

since *the period of oscillation* at each point is given by

$$T = \frac{2\pi}{\omega} = \frac{1}{f}. \tag{4.5}$$

Let us consider the situation when an incident transverse wave moving to the left a having the equation $y_1 = y_0 \sin(\omega t + kx)$ is reflected at the fixed end (see Fig.4.1), producing a new wave propagating to the right and describing by the equation $y_2 = y_0 \sin(\omega t - kx)$.

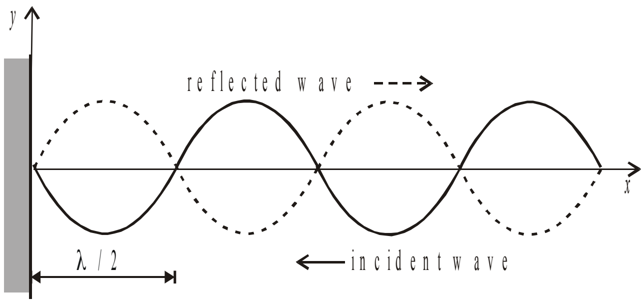

Fig.4.1

The displacement at any point is the result of the interference or superposition of the two waves

$$y = y_1 + y_2 = y_0 \sin(\omega t - kx) + y_0 \sin(\omega t + kx) = \\ = 2y_0 \sin kx \cos \omega t = Y_0 \cos \omega t \quad (4.6)$$

The expression does not represent a travelling wave. It represents a simple harmonic motion whose amplitude varies from point to point, and is given by

$$Y_0 = 2y_0 \sin kx \,. \quad (4.7)$$

Therefore, the eq.(4.6) is the special kind of wave called *standing or stationary wave*. At any point the amplitude is the function of x. From eq.(4.7) follows that the amplitude is zero for $kx = n\pi$, where n=0,1,2,3,... is an integer. This result also be written

$$x = n\frac{\lambda}{2} \,. \quad (4.8)$$

These points are called *nodes* or *nodal points*. The amplitude has a maximum value of $Y = \pm 2y_0$ and these points are known as *the antinodes*. This case occurs when the x is satisfies condition

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots \quad (4.9)$$

or

$$x = (2n+1)\frac{\lambda}{4} \,. \quad (4.10)$$

The results we have obtained find many applications. In acoustic, for example, resonating cavity are used for sound analysis.

Now we shall take interest in the elastic wave in a solid rod. If we produce a disturbance at one end of a solid rod, the disturbance propagates along the rod and eventually is felt the other end. We can show how its velocity is related to the physical properties of the rod. Let us consider a rod of uniform cross section A subject to a stress along its axis indicated by the force F (see Fig.4.2). The normal stress σ at the section of this rod is defined as

$$\sigma = \frac{F}{A}. \tag{4.11}$$

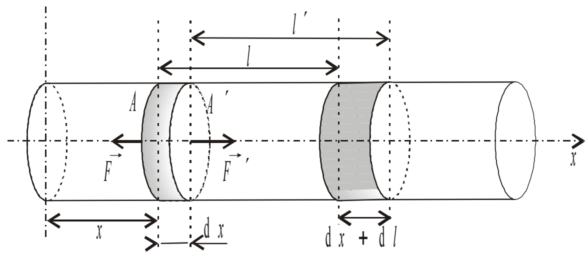

Fig.4.2

Under the action of such forces each sections of the rod suffers a displacement x parallel to the axis, as is shown in Fig.4.2. If the displacement is the same of all points of the rod, there is no deformation. When the forces are applied, the section A is displaced a distance l and section A' a distance l'. The separation between A and A' in the deformed state is then

$$\mathrm{d}x + (l' - l) = \mathrm{d}x + \mathrm{d}l,$$

where $\mathrm{d}l = l' - l$ is the deformation of the rod in that region. Since the deformation $\mathrm{d}l$ corresponds to a length $\mathrm{d}x$ we see that the normal strain in the rod is

$$\varepsilon = \frac{\partial l}{\partial x}. \tag{4.12}$$

Between normal stress σ and the normal strain ε of the rod exists the relation called Hook`s law, which states

$$\sigma = Y\varepsilon, \tag{4.13}$$

where Y is called Young's modulus of elasticity. Introducing eqs.(4.11) and (4.12) into eq.(4.13) and solving for F we get

$$F = YA\frac{\partial l}{\partial x}. \tag{4.14}$$

For the case of a rod, which is not in equilibrium, the force is not the same along the rod. As a result, a section of the rod of thickness $\mathrm{d}x$ is subject to resultant force to the right given by

$$\mathrm{d}F = F - F' = \frac{\partial F}{\partial x}\mathrm{d}x\,. \qquad (4.15)$$

If the mass of section A is $\mathrm{d}m = \rho \mathrm{d}V = \rho A \mathrm{d}x$ and acceleration is $\frac{\partial^2 l}{\partial t^2}$ applying Newton's second law ($\mathrm{d}F = a\mathrm{d}m$) equation (4.15) we may write the equation of motion of the section as

$$\frac{\partial F}{\partial x}\mathrm{d}x = \rho A \mathrm{d}x \frac{\partial^2 l}{\partial t^2}\,. \qquad (4.16)$$

Taking the derivation of eq.(4.14) with respect to x, we have

$$\frac{\partial F}{\partial x} = YA\frac{\partial^2 l}{\partial x^2}\,.$$

Substituting this result in eq.(4.16) gives

$$\frac{\partial^2 l}{\partial t^2} = \frac{Y}{\rho}\frac{\partial^2 l}{\partial x^2}\,. \qquad (4.17)$$

This is the differential equation of motion of the wave propagating along the rod with velocity

$$v = \sqrt{\frac{Y}{\rho}}\,. \qquad (4.18)$$

Next we shall consider elastic waves in a gas resulting from pressure variation in the gas. For simplicity, we shall consider only waves propagated in a gas within a cylindrical tube. There is an important difference between elastic waves in a gas and elastic waves in a solid rod. Gases are very compressible and the density of the gas will suffer the same fluctuations as the pressure.

Let us assume that p_0 and ρ_0 are the equilibrium pressure and density in gas. If the pressure of the gas is disturbed, a volume element $A\mathrm{d}x$ is set in motion because the pressure p and p' on one side and the other are different, giving rise to a net force. The mass within the undisturbed volume element is $\rho_0 A\mathrm{d}x$ and disturbed volume element is $\rho A(\mathrm{d}x + \mathrm{d}l)$, where ρ is the density of the disturbed gas. The conservation of matter requires that both masses must be equal

$$\rho A(\mathrm{d}x + \mathrm{d}l) = \rho_0 A \mathrm{d}x$$

or

$$\rho\left(1 + \frac{\partial l}{\partial x}\right) = \rho_0 .$$

Solving for ρ we have

$$\rho = \frac{\rho_0}{1 + \frac{\partial l}{\partial x}} .$$

Since in general $\frac{\partial l}{\partial x}$ is small, we can replace $\left(1 + \frac{\partial l}{\partial x}\right)^{-1}$ by $\left(1 - \frac{\partial l}{\partial x}\right)$ using Binomial equation, resulting in

$$\rho - \rho_0 = -\rho_0 \frac{\partial l}{\partial x} . \tag{4.19}$$

The pressure is related to the gas density ρ by the equation of state. From it follows that $p = f(\rho)$. For relatively small changes in density we keep only the first two terms in Taylor`s expansion and write

$$p = p_0 + (\rho - \rho_0)\left(\frac{\mathrm{d}p}{\mathrm{d}\rho}\right)_0 . \tag{4.20}$$

The term

$$B = \rho_0 \left(\frac{\mathrm{d}p}{\mathrm{d}\rho}\right)_0 \tag{4.21}$$

is called *the bulk modulus of elasticity*. Inserting this value into eq.(4.20) gives

$$p = p_0 - B\frac{\rho - \rho_0}{\rho_0} . \tag{4.22}$$

This expression corresponds to Hook`s law for gas. Using eq.(4.19) to eliminate $\rho - \rho_0$ we have

$$p = p_0 - B\frac{\partial l}{\partial x} . \tag{4.23}$$

This expression relates the pressure at any point in the gas to the deformation at the same point. We take derivative eq.(4.23) with respect to x and remember that ρ_0 =constant through the gas then

$$\frac{\partial p}{\partial x} = - B\frac{\partial^2 l}{\partial x^2} \tag{4.24}$$

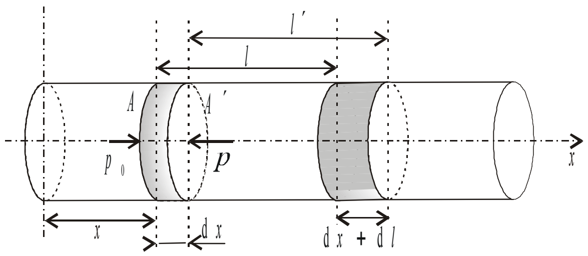

Fig.4.3

Now we derive the equation of motion of the volume element. The gas at the left (see Fig.4.3) of our volume element pushes to the right with a force pA and the gas at the right pushes to the left with a force $p'A$. Then the resultant force in the x-direction is

$$(p - p')A = - A\mathrm{d}p\,, \tag{4.25}$$

where $\mathrm{d}p = p - p'$ and the equation of motion of this element is

$$\frac{\partial p}{\partial x} = - \rho_0 \frac{\partial^2 l}{\partial t^2}\,, \tag{4.26}$$

where $\frac{\partial^2 l}{\partial t^2}$ is acceleration of volume element $\rho_0 A\mathrm{d}x$. If we compare eq.(4.24) and eq.(4.26) we obtain an equation similar to the wave equation. Then the velocity with which the disturbance propagates in a gas as

$$v = \sqrt{\frac{B}{\rho_0}}\,. \tag{4.27}$$

Wave motion in gases in generally an adiabatic process, it means that no energy is exchanged in the form of heat by a volume element of the gas. Under these conditions the pressure of the gas is proportional to ρ^{κ} as

$$p = C\rho^{\kappa}, \tag{4.28}$$

where κ is a quantity characteristic of each gas, C is a constant. We take the derivation of p with respect to ρ as

$$\frac{\mathrm{d}p}{\mathrm{d}\rho} = \kappa C\rho^{\kappa - 1}. \tag{4.29}$$

Using eq.(4.21) we have the expression for bulk modulus of elasticity in form

$$B = \rho_0 \left(\frac{\mathrm{d}p}{\mathrm{d}\rho} \right)_0 = \kappa C\rho_0^{\kappa} = \kappa p_0 .$$

Inserting this value into equation (4.27) and dropping the subscript 0 we find

$$v = \sqrt{\kappa \frac{p}{\rho}} \tag{4.30}$$

or using the equation of state we give

$$v = \sqrt{\kappa \frac{p_0 T}{\rho_0 T_0}} = v_0 \sqrt{\frac{T}{T_0}}, \tag{4.31}$$

where $v_0 = \sqrt{\kappa \frac{p_0}{\rho_0}} = 332\ \mathrm{m \cdot s^{-1}}$ is the velocity of sound under conditions, $T_0 = 273{,}15\ \mathrm{K}$, $p_0 = 1{,}013.10^5\ \mathrm{Pa}$, $\rho_0 = 1{,}293\ \mathrm{kgm^{-3}}$, T is the temperature of air where the velocity of sound is measured.

The method-practical part

A. Measurement of the velocity of sound in air by resonance method

Apparatus is shown in Fig.4.4.

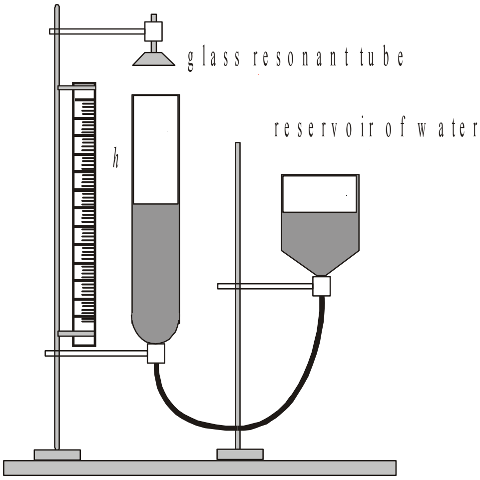

Fig.4.4

Apparatus consists of the tube closed at one end. The length, h, of the air column is varied by moving the tube vertically. Because the tube is closed at one end and open at other , the sound waves generated by the tonal generator are reinforced when the length of the column corresponding to one of the resonant frequencies of the tube. If *the number of resonances at any frequency* detected by oscillograph equals n and h is the distance between first and final resonance then we can calculated the velocity of the sound in air by the relation

$$v = \lambda f = \frac{2h}{n-1} f \tag{4.32}$$

Since the frequency used for loud is known the velocity of sound in air may be calculate from this equation.

Poisson's constant κ can be determined from eq.(4.31). Rewriting of this equation gives

$$\kappa = v^2 \frac{\rho_0 T_0}{p_0 T}, \tag{4.33}$$

where $T = t + T_0$, v is the velocity of sound in air corresponding to the temperature t.

Measurement

Apparatus: glass resonance tube, laboratory stand, water reservoir with rubber tubing for connection to resonance tube, tonal generator. microphone, oscilloscope.

Experimental procedure: Set up the position of microphone at the top glass tube so that it will be barely clear the glass vibrating. Rise the water level in the glass tube until it is near the top. While the microphone is vibrating lower the water level slowly until the first resonance point is reached. Determine the position of the resonant point accurately. In like manner determine the last point of resonance and determine the number of resonances in tube. Repeat the entire process with five different frequencies.

Calculation: From the measurement taken with the resonance tube calculate the value of the velocity of sound in air for each of the five frequencies using eq.(4.32). Determine the mean value of the velocity of sound in air. Compare your experimental value with the correct value of the air at the recorder temperature calculating form eq.(4.31). Calculate the Poisson`s constant given eq.(4.33). Compare your experimental value with the value for diatomic gas $\kappa = 1.4$.

B. Measurement of the velocity of sound in metal rod

Kundt's method of measuring the velocity of sound in metals depends upon setting up standing waves in the air inside a glass tube, which has one end closed. These waves in air are created by longitudinal vibrations in a metal rod, which is positioned axially within the glass tube as is shown in Fig.4.5.

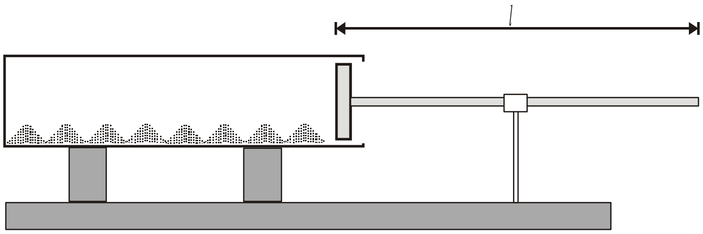

Fig.4.5

The form of the stationary waves in air may be deduced from observation of configurations into which cork dust falls as the air in the tube vibrates in resonance with the natural vibration frequency of the metal tube. The rod itself is stroked longitudinally with the hammer. They may be a variety of cork dust pattern, but regardless of the particular pattern observed, the distance between corresponding points of adjacent configurations will be one-half a wavelength of the sound wave in air.

Measurement

Apparatus: Kundt`s tube, metal rod, cork dust, hammer, meter stick.

Experimental procedure: Measure the length l of metal rod and temperature t of the air. Clamp the test rod exactly at its midpoint. One end of the rod is fitted with the disk and this end is inserted into the long glass tube. The glass tube should be stopped at the other end and should contain enough cork dust to cover lightly the tube. Set the rod in vibration by stroking the free end with hammer. Adjust the position of the glass tube until the best possible cork dust patterns are produced. Measure the distances between the nodal points and record them into data table. The resonance condition is given by

$$f = f'$$

where f and f' are frequencies of oscillations in air and rod, respectively. Using eq. (4.7) we have

$$\frac{v}{\lambda} = \frac{v'}{\lambda'}.$$

Solving for v'

$$v' = v\frac{\lambda'}{\lambda}, \tag{4.34}$$

where $\lambda' = 2l$, note that l is length of the rod.

Calculation: Using eq.(4.34) determine the velocity of the sound in metal rod. Determine the modulus of elasticity from eq.(4.19), where ρ is density of the metal rod. Give a complete analysis of the sources of errors in this experiment.