18. Measurement of the ratio of charge to mass for electron by the method of magnetron.

Assignment

1. Measure the characteristics of magnetron.
2. Calculate the ratio of charge to mass for electron.
3. Calculate the relative error and discuss your result.

Theoretical part

Milikan's - oil drop experiment and innumerable other experiments have shown, that in nature electric charge comes in units of one magnitude, only. That magnitude we denote by *e* and it is called the elementary charge. Its value is approximately equal $e = 1.602 \times 10^{-19}$ C.

One fundamental property of electric charge is its existence in two varieties that were long ago named negative (electron) and positive (positron). Two properties of electronic charge are essential in the electrical structure of matter-charge is conserved and quantized. These properties involved quantity of charge and thus imply a measurement of charge. Electron is the elementary particle, which has also the mass equal approximately $m = 9.109 \times 10^{-31}$ kg.

The ratio of charge of electron to its mass is one of the most important magnitudes in elementary particle physics. Its value can be determined in deflection experiments with beams of electrons moving in both in electric and magnetic fields. In this way $\frac{e}{m_e}$ had been determined by the J. J. Thomson in 1897. This experiment represents discovery of the electron as a fundamentally particle of nature. Its rest mass can be then inferred from value *e* and $\frac{e}{m_e}$.

Simultaneously applied electric and magnetic fields are first adjusted to produce an undeflected beam of charge particles. Total force on the particle carrying charge q and moving with speed v in electric field E and magnetic field B is given by

$$F = F_e + F_m = q(E + v \times B)\,, \tag{2.1}$$

where

$$F_e = qE \tag{2.2}$$

is electric force acting on the charge particle and

$$F_m = q(v \times B) \tag{2.3}$$

is the magnetic force acting on a charge particle in electromagnetic field, q is the charge of particle and v is the speed of the particle. Eq.(2.1) is known as Lorentz's force law.

Rewriting eq.(2.1) and using Newton`s second law give the equation describing the motion of electron in the electromagnetic field in form

$$ma = -e(E + v \times B)\,, \tag{2.4}$$

where a is the acceleration of electron. In generally, the velocity and acceleration of the electron change from point to point and its path will be curved. Eq.(2.4) is the basic equation for determination of ratio of charge to mass for the electron. Its value is $\frac{e}{m} = 1.758 \times 10^{11}$ C.kg^{-1} approximately. This constant can be determined in deflection experiments with beams of electrons moving in the electromagnetic field.

The method-practical part

Eq.(2.4) may be used to measure the ratio of $\frac{e}{m_e}$ for electrons. The measurement of this ratio may be carried out with the device that is called *magnetron*. It consists of the diode in solenoid with cylindrical electrodes, positive anode A of radius a and negative cathode C of radius b. Electrons are emitted and accelerated from the cathode to the anode due to electric field E between these two electrodes.

If the electric field between cathode and anode and magnetic field of solenoid are perpendicular to each other and simultaneously applied on electron beam the electron beam produces deflection. If we measured the geometry of the magnetron and values of E and B the ratio of $\frac{e}{m_e}$ may be determined.

If we place the cylindrical diode into magnetic field created by the long solenoid so that the axis is parallel with a magnetic lines then the direction of moving electron leaving the cathode are perpendicular to magnetic line.

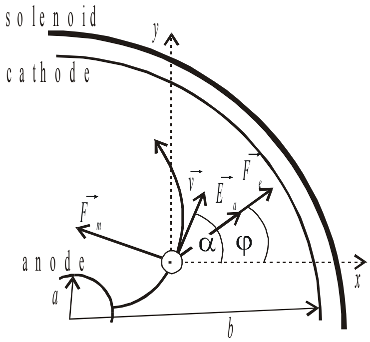

Fig.2.1

From Fig.2.1 we can see that there are two forces acting on the electron, electric force $F_e = eE$ and magnetic force $F_m = evB$ since the angle between v and B equals 90° (see eq.2.3)). The speed v of electrons emitted from cathode may be calculated from the law of conservation of energy as

$$eU_a = \frac{1}{2} m v^2$$

where U_a is a potential difference between anode and cathode.

The components forms of equation of motion are

$$m \frac{d^2 x}{dt^2} = eE_{ax} + eBv_x \tag{2.5}$$

$$m \frac{d^2 y}{dt^2} = eE_{ay} + eBv_y , \tag{2.6}$$

where E_{ax} and E_{ay} are electric field components between anode and cathode. The relation between potential difference U_a and electric filed E_a by the definition equals

$$U_a = -\int_a^b \vec{E} . d\vec{r} .$$

From this expression follows that E_a is inversely proportional to the distance between the electrodes. The Fig.2.1 shows the magnitudes of components of electric field vector E and speed v at any point P:

$$E_{ax} = E_a \cos\varphi = E_a \frac{x}{\left(x^2 + y^2\right)^{1/2}} \tag{2.7}$$

$$E_{ay} = E_a \sin\varphi = E_a \frac{y}{\left(x^2 + y^2\right)^{1/2}} \tag{2.8}$$

$$v_x = v\cos\alpha = \frac{dx}{dt} \tag{2.9}$$

$$v_y = v\sin\alpha = \frac{dy}{dt} . \tag{2.10}$$

If we insert these equations into eqs.(2.5), (2.6), we give

$$m \frac{d^2 x}{dt^2} = eE_{ax} + eB \frac{dx}{dt} \tag{2.11}$$

$$m \frac{d^2 y}{dt^2} = eE_{ay} + eB \frac{dy}{dt} . \tag{2.12}$$

Solving eqs.(2.11), (2.12) for constant U_a and various magnitudes of B we give the trajectory of electrons in form of cycloid.

This physical principle may be used to measure the ratio of e/m_e. The apparatus is shown in Fig.2.3. Electrons are accelerated by the electric field between cathode and anode. If this diode is located in the perpendicular magnetic field created by the current I_s passing through the solenoid, the enhancement of magnetic field produces deflection of electrons. During this process we observe the reduction of current in anode curcuit I_a. Typical graph of I_a versus I_s is in Fig.2.2.

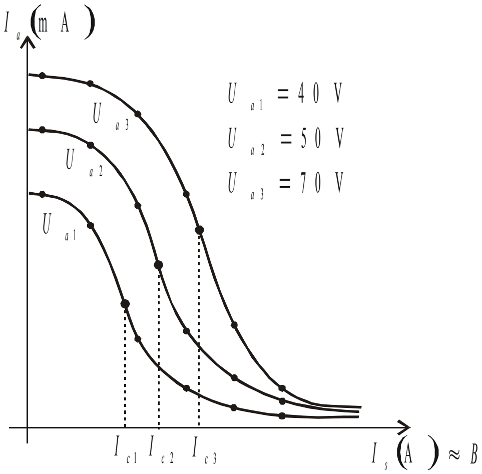

Fig.2.2

From the figure we can see that for the certain value of magnetic field that is called *critical magnetic field* the anode current falls rapidly to zero. If we solve the expression (2.11) and (2.12) for the critical value of B_c we give

$$\frac{e}{m_e} = \frac{2U_a}{B_c^2 b^2 \left(1 - \frac{a^2}{b^2}\right)}, \tag{2.13}$$

where the critical value of B_c inside the solenoid is given by the expression

$$B_c = \mu_0 \frac{NI_c}{2l} \cos\alpha \tag{2.14}$$

and $\mu_0 = 4\pi \times 10^{-7}$ N.A^2 is the permeability of free space, N is the number of turns of solenoid, l is the length of solenoid, $\alpha = \arctan \frac{D}{l}$ and D is the diameter of solenoid. Inserting this value into eq.(2.13) we give

$$\frac{e}{m_e} = \frac{8l^2}{N^2 \mu_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right) \cos^2 \alpha} \frac{U_a}{I_c^2} \qquad (2.15)$$

From this expression we can see that the first term depends on the geometry only. Therefore, $\frac{e}{m_e}$ depends linearly on $\frac{U_a}{I_c^2}$.

Note: the cathode is obviously made by the metal wire with the small radius a. In this case $\frac{a^2}{b^2} \approx 0$ and eq.(2.15) is in form

$$\frac{e}{m_e} = \frac{8l^2}{N^2 \mu_0^2 b^2 \cos \alpha} \frac{U_a}{I_c^2} \qquad (2.16)$$

Measurement

Apparatus: diode placed into solenoid, two ammeters, two potentiometers, source of voltage.

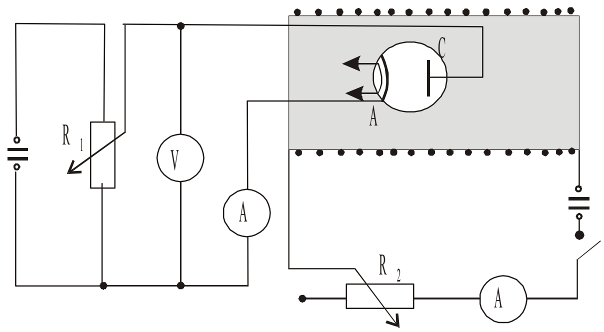

Fig.2.3

Experimental procedure: Circuit diagram for the electric connection is shown in Fig.2.3. Connect apparatus and heat the cathode. Set the voltage U_a between cathode and anode. Measure the current I_a for falling values of I_s for three different values of voltage U_a.

Calculation: Plot the graph I_a versus I_s. Determine the critical values of I_s (inflexion point on the graphs) for each value of voltage U_a. Draw graph I_c^2 versus U_a and calculate the value of $\frac{e}{m_e}$ using eq.(2.15). Compare your result with the correct value and analyze the source of errors in your experiment.