17. CONDENSER CHARGING/DISCHARGING PROCESS ANALYSIS

Objectives:

1. Do a quantitative analysis of the condenser charging process. Check the condenser charging time dependency during its charging. Determine the circuit's time constant <u>RC</u> and the starting voltage of the condenser charging.
2. Measure the condenser discharge time dependency and determine the circuit's time constant <u>RC</u>. Calculate the condenser capacity using a known resistor.

Theoretical Introduction

Let's imagine we have an electric circuit containing a condenser of the capacity <u>C</u> which charges itself through a resistor with a known resistance <u>R</u> from an electricity source with a constant voltage $\underline{U_0}$ (pic 17.1). The voltage <u>U</u> on the condenser with a charge <u>Q</u> can be derived from the condenser capacity formula

$$C = Q/U$$

According to Kirchhoff's law for this closed electric circuit

$$U_0 - U = RI \qquad (17.1)$$

is valid, where I is the charging current of the condenser, that is when

$$I = dQ / dt = C\, dU / dt \,. \qquad (17.2)$$

Combining these two equations we receive the following equation

$$U_0 - U = RC\, dU / dt \qquad (17.3)$$

wherefrom we can derive

$$dt / RC = dU / (U_0 - U) \qquad (17.4)$$

after separating the variables. Characteristics of a circuit in this equation are represented by a product of the capacity and resistance $\tau = RC$. The unit of this product has a value with a time dimension so it's called the <u>time constant of a circuit.</u> Eq. (17.4) is dimensionless, that means that only quantities of the same dimensions occur in this equation. We can receive

$$- t / RC = \ln(U_0 - U) + K \qquad (17.5)$$

after its integration. We determine the integration constant <u>K</u> from the initial conditions. Initial voltage (time t = 0) on the condenser will be marked $\underline{U_p}$. We will receive

$$- t / RC = \ln (U_0 - U) - \ln (U_0 - U_p) \qquad (17.6)$$

after substitution for $K = -\ln(U_0 - U_p)$ into eq. (17.5). Now we can express the voltage on the condenser using the following equation

$$U = U_0(1 - e^{(-t/RC)}) + U_p e^{(-t/RC)} . \qquad (17.7)$$

We can check the validity of eq. (17.7) or (17.6) and determine the time constant by measuring the voltage U and time t.

Kirchhoff's laws for electric circuits

$$\varepsilon = IR \qquad (17.8)$$

determine the time dependency of condenser discharging i. e. dependency of the electric current or voltage on the condenser in an RC circuit. It might be useful to realize that the electric current I is the decrease of the electric charge Q on the condenser during a time period

$$I = dQ / dt$$

and the electromotoric voltage U_e is a ratio of the charge Q on the condenser and its capacity C. After substituting into eq. (17.8) we get a differential equation we solve by separating the variables

$$Q/C = - dQ/dt * R \;\Rightarrow\; dQ/Q = - dt/RC \;\Rightarrow\; \ln Q = - t/RC + \ln K .$$

After simplification for the voltage on the condenser $U = Q / C$ we get

$$U = K/C * e^{(-t/RC)} = U_0\, e^{(-t/RC)} = U_0\, e^{(-t/\tau)} \qquad (17.9)$$

where U_0 is the initial voltage and the task is to determine the time constant of the circuit

$$\tau = RC \qquad (17.10)$$

Measurement method – task 1

The condenser won't be charged by the electric source to its voltage U_0 but only to the predefined voltage U_z. We will determine the time needed for charging the condenser to this predefined voltage U_z by a spontaneously discharging circuit. A suitable block scheme of the measuring device is shown in the picture 17.2. When the voltage U reaches U_z on the condenser, the comparison circuit discharges the condenser to voltage U_p during a time period about 10 000x smaller than the charging time is, however we are not talking about a thorough discharge of the condenser to a zero voltage. The condenser starts to recharge again from this voltage.

The discharging voltage may cause a short detectable flash when passing through a LED diode. These flashes appear consecutively in time intervals needed for charging/discharging of the condenser. Neglecting time used by discharging vs. charging we can say that the time period between two LED flashes is period t_z of the charging of the condenser to the predefined voltage U_z from U_p. There's the time development of the voltage shown in the picture 17.3. The dashed curve represents the charging of the condenser from $U = 0V$ to U_0 and the continuous line represents the process in the spontaneously discharging circuit.

pictures…

Measurement – task 1

Apparatus: condenser, resistor, proceeding and comparison circuit, voltmeter, stopwatch, conductors.

Procedures: Connect the resistor and condenser to the labeled clamps of the comparison circuit (3, 4, 5). First attach the voltmeter to the clamps (3, 5) of the source of a constant electric voltage U_0 and check the readiness of the set. Connect the voltmeter to the clamps (1, 2) of the variable voltage U_z after reading the value of voltage U_0. Select a voltage U_{zi} using the potentiometer R_3. The time period of the condenser discharging t_{zi} will be determined by the stopwatch measuring the time between more LED flashes and afterwards dividing this value by the number of flashes. Repeat the measurement with different values of the voltage using the whole voltage interval of the potentiometer R_3.

Elaboration of the measured values: Analyse the measured charging process of the condenser according to the eq. (17.6) where t is the time of the condenser charging t_z and the voltage U is the maximum voltage before the discharge of the condenser U_z.

$$\ln (U_0 - U_z) = - t_z/RC + \ln (U_0 - U_p) \qquad (17.11)$$

rewrite it in a linear equation form $y = ax + b$ (17.12)

where $x = t_z$, $y = \ln (U_0 - U_z)$. Substitute for $x_i = t_{zi}$ and $y_{zi} = \ln(U_0 - U_{zi})$ for the i-th measurement. Values measured by this way should lie on one line therefore we'll use the linear regression for evaluation of the results of measurements. Make sure that the linear regression is a suitable method to relay the curve by the measured values and thereby checking the validity of (17.6), determine the constants for this regression.

Physical contents of parameters a and b is apparent from the comparison of eq. (17.11) and (17.12). The slope of line b and the intercept on O_y – a is

$$b = - 1/RC , \qquad a = \ln (U_0 - U_p) . \qquad (17.13)$$

You'll determine the voltage U_p using the calculated constants (17.13) i. e. the initial voltage of the charging or the rest of the voltage which stays on the condenser after its discharge and the time constant of the circuit τ.

You'll determine the time constant $\tau = RC$ from the nominal values of the resistor resistances and from the capacity of the condenser. Compare the value of τ determined in this way with the measured value. Regression analysis may be done on the computer and then the coefficient of correlation for this dependency (17.12) shall be determined.

Measurement method – task 2

The condenser will be charged by the source. Time development of its discharge after being disconnected from the electric source and after being connected to the resistor will be measured with the help of the IP Coach. We'll determine the time constant $\underline{\tau}$ by comparing eq. (17.9) with a numerical fitting of an exponential function

$$y = ae^{bx} + c \; . \qquad (17.14)$$

Measurement – task 2

Apparatus: condenser, resistor, measuring device IP Coach.

Procedures: Integrate the condenser and the resistor into the electric circuit according to the picture 17.4 which shows the suitable scheme, wherefrom you should understand the problem. Select PROGRAMY → MULTISKOP in the program IP Coach. Choose and select the suitable parameters in the section NASTAVIŤ → MERANIE: select DOBA MERANIA . The measurement will start itself automatically after every voltage value decrease from the maximum value. For this reason select SPÚŠŤACIA HRANA → DOLE and set the SPÚŠŤACIU ÚROVEŇ below the value of maximum voltage on the condenser (pic 17.5). First charge the condenser to the max. acceptable voltage with the help of a switch. Switch the program MERAŤ, initialize the measurement with the command ŠTART, switch the switch from the charging position to the position connecting the resistor. The measurement will start itself automatically after a decrease of the voltage to the starting level. Select the step NÁVRAT (ends the program of measuring). Select the step SPRACOVANIE → ANALYZOVAŤ → FIT FUNKCIE, exponential function $y = ae^{bx} + c$ and the step NUMERICKÉ FITOVANIE. End the fitting and write down the measured constants $\underline{c}$, $\underline{b}$, $\underline{c}$ in the moment when the fitting function overlaps the measured one. Return to the process MULTISKOP through the program NÁVRAT. Change the parameters NASTAVIŤ suitably and repeat the whole process with other values (change the DOBA MERANIA and the SPÚŠŤACIA ÚROVEŇ). Repeat the measurements with resistors with different values of resistance.

Note: The fitting program doesn't have to find the correct parameters by itself especially if the time interval is too long. In this case it's needed to do a selection of values in such a way, so that the voltage decreases only to a tenth of the previous voltage within the range of the measured points. If you already know the parameters of the fitting function $\underline{a}$, $\underline{b}$, $\underline{c}$, it is possible to type to the following numerical fitting as the first approximation. In this case the program will find new parameters also for a longer time interval.

Elaboration of the measured values: Determine the average value of the constant $\underline{b}$ and its uncertainty from the set of measurements. Determine the time constant $\underline{\tau}$ by comparing eq. (17.9) and (17.14)

$$\tau = - 1/b \qquad (17.15)$$

If we compare this value with time constant calculated from the data about the condenser capacity and the resistor resistance, there may be a great difference between these two values. It may be caused by a possible equality of the entering resistance $\underline{R}_v$ of the measuring circuit with the resistance of the RC circuit by itself. The determined time constant corresponds to the total resistance $\underline{R_c}$ for which the following characteristic is valid

$$1/R_c = 1/R + 1/R_v \quad => \quad \tau = R_cC = RR_v / (R+R_v) \quad * \; C = \tau' * R_v /(R+R_v) \qquad (17.16)$$

where the time constant of the RC circuit is $\underline{\tau}' = RC$.
These constants are identical only for $\underline{R_v} >> \underline{R}$
Calculate the capacity of the condenser from the known resistance $\underline{R}$ and $\underline{R_v}$ according to the formulas (17.15, 17.16.)

$$C = - 1/R_cb = - (1/R + 1/R_v)\, 1/b \qquad (17.17)$$

We'll determine the uncertainty of the capacity of the condenser from an estimate of resistances and the calculated constant $\underline{b}$. Compare the measured capacity with the one stated on the condenser.

If you have time, measure the time constant of the circuit, for which the resistance of the resistor is much smaller than the entering resistance of the measuring circuit ($R < 0.1\, R_v$).
….just for comparison…