14. Measurement of the temperature dependence of the resistance of conductor. Determination of the temperature coefficient of resistivity.

Assignment

1. Measure the temperature dependence of the resistance.
2. Determine the resistance at reference temperature.
3. Determine the temperature coefficient of resistivity.
4. Analyze the source of errors in your experiment.

Theoretical part

We know that substances are classified in terms of their ability to conduct electrical charges as conductors, semiconductors and insulators. The enormous variation in electrical conductivity of these materials may be explained in terms of energy bands. In this case we describe a classical model of electrical conduction in metals.

Consider a conductor as a regular array of atoms containing free electrons (conduction electrons). These are free to move through conductor and are approximately equal in number to the number of atoms. In the absence of an electric field, the free electrons move with average speeds of order of $10^6\ \mathrm{m.s^{-1}}$ in random motion through conductor. (These speeds may be calculated only if we use the principle of quantum physics.) This situation is similar to the motion of gas molecules confined in vessel. These electrons in a metal are refered as *an electron gas* and they are not totally free since they undergo frequent collisions with the array of atoms. These collisions are the important mechanism for the resistivity of a metal at normal temperature. In the absence of an electric field there is no current through the conductor. If the electric field is applied to the metal, the free electrons drift in the direction opposite that of the electric field with an average drift speed v_d, which is much smaller ($\sim 10^{-4}$ m.s^{-1}) than the average speed between collisions ($\sim 10^{6}$ m.s^{-1}). The electrons in the electric field lost the energy in the collision process and they give up the energy to the atoms in the collisions. Due to this process the energy transformes into vibrational energy of atoms.

Let us consider a conductor of cross-sectional area A carrying a current I. *The current density* j in the conductor is defined to be the current per unit area. Since

$$I = nqv_d A,$$

where n is the number of electrons and q is the charge of electron then the current density is given by

$$j = \frac{I}{A} = nqv_d, \tag{1.1}$$

where n is the number of charges, q is charge of the particle and v_d is the drift velocity of the charges. This expression is valid only if the current density is uniform and the surface is perpendicular to the direction of the current. In general, the current density is a vector quantity given by

$$j = nqv_d \tag{1.2}$$

From this equation we can see that current density is in direction of the motion of the charges for positive charge carriers and opposite to the direction of motion for negative charge carries.

In many cases, the current density in a conductor is proportional to the electric field E in the conductor. That is

$$j = \sigma E, \tag{1.3}$$

where σ is called *the conductivity of the conductors*. Materials that obey eq.(1.3) are called *ohmic materials*.

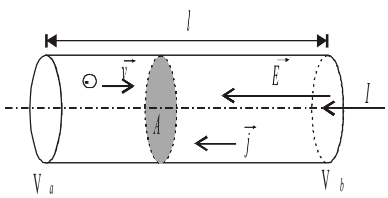

Fig.1 1

If we assume the electric field in the wire is uniform then from Fig.1.1 the potential difference $V = V_B - V_A$ is related to the electric field through the relationship following from the definition of potential difference

$$V = V_b - V_a = -\int_a^b \vec{E} \cdot \mathrm{d}\vec{l} = E\int_0^l \mathrm{d}x = El\,, \tag{1.4}$$

where l is the length of the wire. Inserting eq.(1.4) into eq.(1.3) gives

$$j = \sigma\frac{V}{l}\,. \tag{1.5}$$

Since $j = \frac{I}{A}$, the potential difference can be written

$$V = j\frac{l}{\sigma} = \frac{l}{\sigma A}I\,. \tag{1.6}$$

The quantity

$$\frac{l}{\sigma A} = R \tag{1.7}$$

is called *the resistance* of the conductor. Then

$$R = \frac{l}{\sigma A} = \frac{V}{I}\,. \tag{1.8}$$

This relation is known as *Ohm`s law*. The resistance has SI units of volts per ampere. One volt per ampere is defined to be ohm, Ω. The inverse of the conductivity of material is called *the resistivity* , ρ :

$$\rho = \frac{1}{\sigma}\,. \tag{1.9}$$

Inserting this expression into eq.(1.7) the resistance can be expressed as

$$R = \rho\frac{l}{A}\,. \tag{1.10}$$

The unit of resistance is Ω.m. In according to this classical model, the conductivity and resistivity do not depend on the electric field.

The resistivity of conductor depends on a number of factors, one of which is temperature. The resistivity of a conductor varies in an approximately linear fashion with temperature over a limited temperature range according to the expression

$$\rho = \rho_0 (1 + \alpha \Delta T), \tag{1.11}$$

where ρ is the resitivity at some temperature T, ρ_0 is the resistivity at some reference temperature T_0 and α is called *the temperature coefficient of resistivity*. From eq.(1.11) we have

$$\alpha = \frac{1}{\rho_0} \frac{\Delta \rho}{\Delta T}, \tag{1.12}$$

where $\Delta \rho = \rho - \rho_0$ is the change of resistivity in the temperature interval ΔT.

Since the resistance of a conductor is proportional to the resistivity, eq.(1.10), the temperature variation of the resistance can be written as

$$R = R_0 (1 + \alpha \Delta T) = R_0 + R_0 \alpha \Delta T. \tag{1.13}$$

From this equation *the temperature coefficient of resistivity* equals

$$\alpha = \frac{1}{R_0} \frac{\Delta R}{\Delta T}, \tag{1.14}$$

where $\Delta R = R - R_0$ is the change of resistance in the temperature difference ΔT.

From eq.(1.13) we van see that the resistance varies linearly with temperature over a limited temperature range. The curve is linear over a wide range of temperatures and R increases with increasing temperature (Fig.1.2). Every ohmic material has a characteristic value of resistivity ρ, a parameter depends on the properties of the material and on the temperature, too. In linear region of ρ the ohmic materials, such a cooper ($\rho = 1.7 \times 10^{-8}$ Ω.m at $20\,^\circ\mathrm{C}$), have a linear current-voltage relationship over a large applied voltage, see eq.(1.8), as is shown in Fig.1.3.

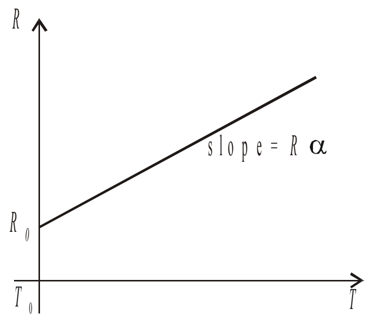

Fig.1 3

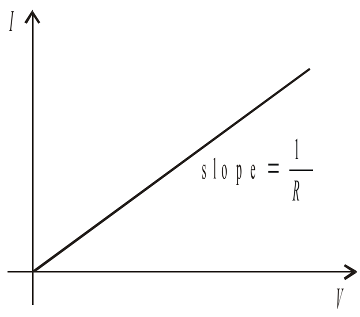

Fig.1 2

The method-practical part

The precise measurement is very important for the determination of the dependence of the resistance versus temperature. One of many methods, which are used for the measurement of resistance is the method known as a Wheatstone bridge method. The circuit consists of the unknown resistor R_x, three known resistors R_1, R_2 and R_3, galvanometer, and the source of emf. This circuit is shown in Fig.1.4.

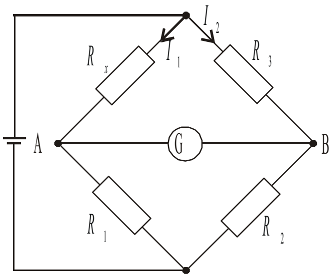

Fig.1 4

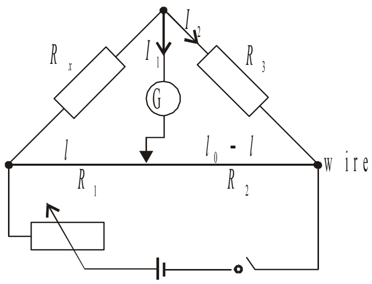

Fig.1 5

The known resistors R_1 is varied until galvanometer reading (G) is zero, that is, until there is no current from point A to point B. Under this condition the bridge is said to be balanced. Since the potential at point A must equal the one at point B, when the bridge is balanced, the potential difference across R_1 must equal the potential difference across R_2. Likewise, the potential difference across R_3 must equal the potential difference R_x. From this follows that

$$I_1 R_x = I_2 R_3$$
$$I_1 R_1 = I_2 R_2 .$$

Dividing first equation by second equation and solving for R_x we find

$$R_x = \frac{R_1}{R_2} R_3 \tag{1.15}$$

From this equation follows the method of measurement of temperature dependence of the resistance R_x since resistances R_1, R_2 and R_3 are known quantities. The resistances R_1 and R_2 are realised by the homogeneous resistor wire of the constant cross-sectional area A. The length of the wire has value of l_0. This wire is stretched on the wooden plate with calibrated meter. The Wheatstone bridge is balanced with the help of the rider. The position of the rider divides the resistor wire into two parts of the length l and $l_0 - l$. Then the values of the resistors can be calculated from the expression

$$R_1 = \rho \frac{l}{A} \qquad R_2 = \rho \frac{l_0 - l}{A} \tag{1.16}$$

Inserting eqs.(1.16) into eq.(1.15) gives

$$R_x = \frac{l}{l_0 - l} R_3 \tag{1.17}$$

where R_3 is known resistor, which value is chosen so the bridge could be balanced. Situation is shown in Fig.1.5.

Measurement

Apparatus: Wheatstone bridge, source of voltage (emf), potentiometer, resistor, device for the heating of the resistor, thermometer.

Experimental procedure: Circuit diagram for the electric connection is shown in Fig.1.5. Connect the apparatus. Balance the Wheatstone bridge and record the value of t_i and the corresponding value of l_i into table Tab.1.1. Repeat the measurement for various temperatures in temperature interval up to $70\,^\circ\mathrm{C}$.

i	t_i	l_i	R_{xi}

Tab.1.1

Calculation: From eq.(1.17) calculate the values of R_x for different temperatures. Draw the graph of resistance versus temperature. Calculate the temperature coefficient of resistivity using the values of R_0 and $R_0\alpha$ determined by the linear regression. Analyse the sources of errors in your experiment.