12. MEASUREMENT OF THE POISSON CONSTANT THE METHOD CLEMENT-DESORMES METHOD

ASSIGNMENT

1. Measure the ratio of C_P/C_V
2. Calculate the error of the measurement

THEORETICAL PART

Substances differ from one to another in the quantity of heat needed to produce a given rise of temperature in a given amount of the gas. The amount of heat Q required to change the temperature of a system is found to be proportional to the mass m of the system and the temperature change ΔT. This can be expressed by the equation

$$Q = mc\Delta T \tag{10.1}$$

where c is a quantity characteristic of the material and it is called **the specific heat.** The values of the specific heats depend to some extend on the temperature, but for a small temperature changes c can be often considered constant. The specific heat depends also on the kind of processes heating. In eq.10.1 we assumed the process was carried out at the constant pressure-atmospheric pressure. For such a process we usually call c **the specific heat at constant pressure.** and it is denote by the symbol c_P. If the volume of the material is kept constant, then specific heat is called as **specific heat at constant volume** and it is denote by the symbol c_V. For the solids and liquids the difference between c_P and c_V is small and we can write

$$c_P = c_v = \frac{1}{m}\frac{Q}{\Delta T} = \frac{1}{m}\frac{dU}{dT} \tag{10.2}$$

However the difference of gases between c_P and c_V are quite different This may be explain in terms of the first law of thermodynamics and the kinetic theory of gases
We introduce **the molar heat capacity at constant volume** as

$$C_v = \frac{1}{n}\left(\frac{dQ}{dT}\right)_V \tag{10.2}$$

and **the molar heat capacity at constant pressure** as

$$C_P = \frac{1}{n}\left(\frac{dQ}{dT}\right)_P \tag{10.3}$$

which are defined, as the heat required raising 1mol of the gas by 1K at constant volume and constant pressure.

We shall imagine that the gas is going from one state to another state quasistatistically, by which we mean that the process is carried out extremely slowly through a succession of infinitesimally close equilibrium states. The change in internal energy for an ideal gas can be expressed as

$$\Delta U = nC_v\Delta T \tag{10.4}$$

where C_v is the molar heat capacity of the gas at constant volume, n is the number of moles and

ΔT is the change in the temperature between two states of a gas.
In the limit of differential changes we can use the first law of thermodynamics to express the molar heat capacity in the form

$$C_v = \frac{1}{n}\frac{dU}{dT} \tag{10.5}$$

The internal energy of the monatomic gas equals

$$U = \frac{3}{2}nRT \tag{10.6}$$

Inserting this equation into eq.10.5 gives the value of the molar heat capacity at constant volume as

$$C_v = \frac{3}{2}R \tag{10.7}$$

where R is the universal gas constant

Now suppose that the gas is transferred to the system at constant pressure. Let the temperature increases by ΔT. The heat must be transferred to the gas. Its value is given by $Q = nC_pT$. Since the volume increases in this process, the work done by the gas is $W = PdV$. Applying **the first law of thermodynamics** we get

$$\Delta U = Q - W$$

or

$$nC_V\Delta T = nC_p\Delta T - nR\Delta T \tag{10.8}$$

If you can see from this expression the part of the heat transferring to the system increases the internal energy of the system and second part is used to do external work by moving a piston. From this expression follows the very important relation between the molar heat capacity at constant pressure and constant volume as

$$C_p = C_v + R \tag{10.9}$$

If you can see from this expression the molar heat capacity at constant pressure is grater than the molar heat capacity at constant volume by an amount of the universal gas constant. We introduce the a new physical constant

$$\gamma = \frac{C_P}{C_V}$$

This constant is dimensionless quantity and it is called **the Poisson constant**. Its value depends on the number of atoms in the molecule. For an monatomic gas, for example, the value of this constant is

$$\gamma = \frac{5}{3} \tag{10.10}$$

The values of C_P and χ are in excellent agreement with experimental values for monatomic gases. The internal energy and hence the molar heat of a complex gas must include contributions from the rotational and vibration motion of molecule. The rotational and vibration motion of molecules with structure can be activated by collisions and therefore are „coupled" to the translation motion of molecules. The statistical mechanics has shown that for a large number of particles the available energy is, on the average, shared equally by each independent degree of freedom. The equipartition theorem states that at equilibrium each degree of freedom contributes, on the average, $\frac{1}{2}kT$ of energy per molecule. Then for the i degree of freedom we have
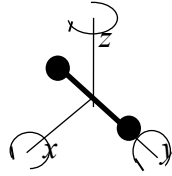
$$C_v = \frac{i}{2}R \tag{10.11}$$

and

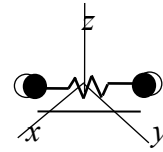$$\chi = \frac{C_p}{C_v} = \frac{i+2}{i} \qquad (10.12)$$

For a diatomic molecule is the situation shown in Fig.10.1
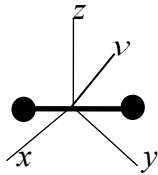
translation motion
of the centre of mass

rotational motion
about various axis

vibration motion
along the molecular axis

For this molecule we can neglect the rotation about y - axis since the moment of inertia and the rotational energy $\frac{1}{2} I \omega^2$ about this axis are negligible compared with those associated with the x and z-axis. Thus there are five degrees of freedom- three degrees of freedom associated with the translation motion and two associated with the rotational motion. Then from eq. (10.11) we have

$$C_v = \frac{3}{2} R \qquad (10.13)$$

Therefore, the value of Poisson's equation for diatomic molecules equals

$$\chi = \frac{5}{2} = 1.40 \qquad (10.14)$$

If an ideal gas undergoes an adiabatic expansion or compression, the first law of thermodynamics together with the equation of state shows that

$$PV^\chi = const \qquad (10.15)$$

Equation 10.15 is called **the Poisson equation for a adiabatic process.**
Using this equation for a gas that transfers from the initial state to another final state we have

$$P_i V_i^\chi = b V_f^\chi \qquad (10.16)$$

From this equation follows

$$\left(\frac{V_f}{V_i}\right)^\delta = \frac{P_i}{b} \qquad (10.17)$$

The ratio of the initial and final volume may be calculated by the using the Boyle law as

$$\frac{V_f}{V_i} = \frac{P_i}{P_f} \qquad (10.18)$$

Then the ratio of heat capacities χ can be wrote as

$$\chi = \frac{\log P_i - \log b}{\log P_i - \log P_f} \qquad (10.19)$$

THE METHOD - PRACTICAL PART

Clement and Desormes in 1819 proposed the simple method of measurement two important constants of gases: the molar heat capacity C_p at the constant pressure and the molar heat capacity C_v of the gas at the constant volume. The device for this measurement is shown in Fig.10.2.
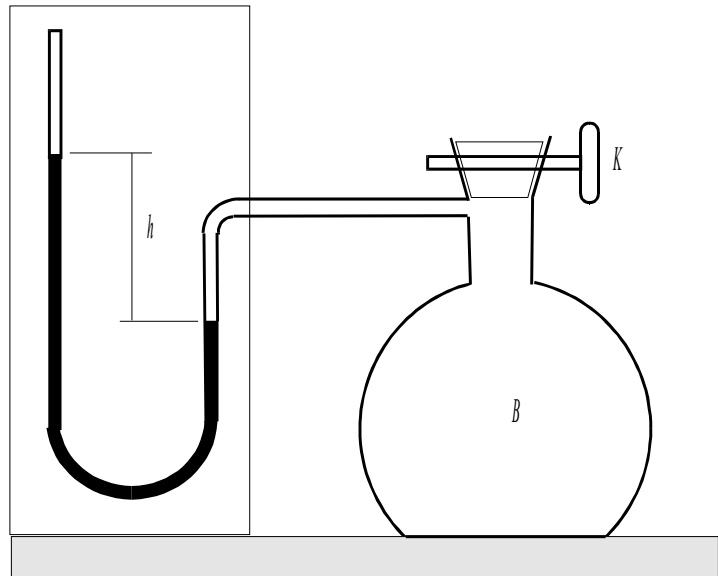
In a bulb B is closed a gas. The open-tube manometer measures the pressure inside the bulb. Then the pressure of gas (air) in bulb is equal

$$P = b + \rho g h$$

(10.20)

where b is atmospheric pressure, ρ is the density of the liquid, h is the difference between the heights of the liquid .in the arms of U-manometer and g is acceleration of gravity

The method is based on the measuring the pressure P_1 of the air in the bulb until enhanced the pressure through the stopcock K_1 and the pressure of the air if the pressure in the air in the bulb decreases after the stopcock K_2 is quickly turn round. Pump the air through the stopcock K_1 into the bulb and read the difference between the columns of the liquid in manometer. Quickly turn round the stopcock K_2 .If the air in the bulb is in thermal equilibrium measure the difference between the column of liquid in manometer again.

Note the process must be adiabatic, i.e. no heat enters or leaves the system ($\Delta Q = 0$). An adiabatic process can be achieved by the performing the process rapidly.

MEASUREMENT

Apparatus: Clement-Desormes apparatus, barometer, thermometer.

We shall measure the Poisson constant of the air. The air consists of 78% of nitrogen, 21% of oxygen and 1% of rare gases. All gases are diatomic ones. Therefore, the Poisson constant with respect to eq.10.14 has the value of 1.40.
Measure the atmospheric pressure b by the barometer. Measure the temperature t. Measure the pressure of the air in the bulb by the method that is described above. Repeat these measurements a few times and record them into Tab.10.1.

CALCULATION

Using eq.10.20 calculate the pressure P_i and P_f for every measurement. Using eq.10.19 calculate the Poisson constant γ for each pair of P_i and P_f. Remember that the pressure P_i is the pressure of the air after the pumping the air into the bulb and P_f is the pressure of the air if the air is loose from the bulb. Calculate the percentage error of measurement as

$$\frac{\Delta y}{y_t} \cdot 100\%$$

where $\Delta y = y_m - y_t$, $y_t = 1 \cdot 40$.

Analyse the source of error in this experiment