9. MEASUREMENT OF THE ELASTICITY OF SOLIDS DETERMINATION OF THE TENSILE STRENGTH AND YOUNG MODULUS

OBJECTIVES

1. Measure the tensile strength of the elastic wire
2. Determine the Young's modulus of the elastic wire
3. Calculate the relative error of Young's modulus

THEORETICAL PART

Every substance is elastic to some degree. A material is said to be elastic if after a deformation of any kind it returns readily to its original shape. Through the application of external force on an object changes the shape or rise of the object (or both). These changes are observed as large-scale deformations but the internal forces that resist the deformation are due to short-range forces between atoms.

When the external force *F* is applied along the elastic wire perpendicular to the cross section *A*, internal force in the wire resists distortion but wire attains an equilibrium in which the length is greater and in which the external force is balanced by internal force.

We know that **the tensile stress** σ is defined as the ratio of the magnitude of the external force *F* to the cross-sectional area *A*

$$\sigma = \frac{F}{A} \tag{1}$$

The unit of this magnitude is N m^{-2} (Pa).

We define **the tensile strain** ε as

$$\varepsilon = \frac{\Delta l}{l_0} \tag{2}$$

where the change in length after application the force is $\Delta l = l - l_0$ and l_0 is the original length of the wire. Remember that the tensile strain is the dimensionless quantity

Hook's law states that the tensile stress is directly proportional to the tensile strain as

$$\sigma = Y\varepsilon \tag{3}$$

where the constant of proportionality *Y* is called **the Young modulus of elasticity**. From eq.3 follows that

$$Y = \frac{\sigma}{\varepsilon} \quad (4.4)$$

Inserting eqs.1 and 2 into eq.4 gives

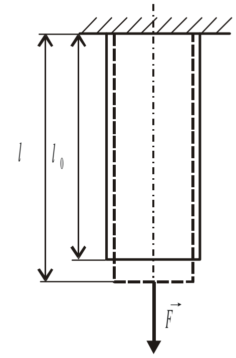

Fig.1

$$Y = \frac{Fl_0}{A\Delta l} \quad (4.5)$$

as is shown in Fig.1.

This quantity has dimension of Newton divided by m ^{2} , or Pa. Its value is typically used to characterize a rod or wire stressed under either tension or compression.

Experiments show that

1. The change in the length for a fixed applied force is proportional to the original length
2. The force necessary to produce a given strain is proportional to the cross-sectional area.
3. Graph of the stress as function of strain is illustrated in Fig.2

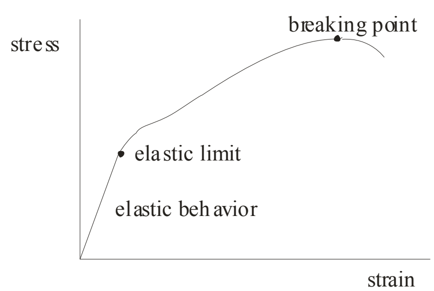

Fig.2

From the Fig.2 we can see that when the stress exceeds the elastic limit, the object is permanently distorted and will not return to its original shape. Beyond the elastic limit, the stress-strain curve departs from a straight line.

As the stress is increased even further, the material will untimely break.

THE METOD-PRACTICAL PART

Laboratory support using for the measurement of the elasticity of the wire is shown in Fig.3

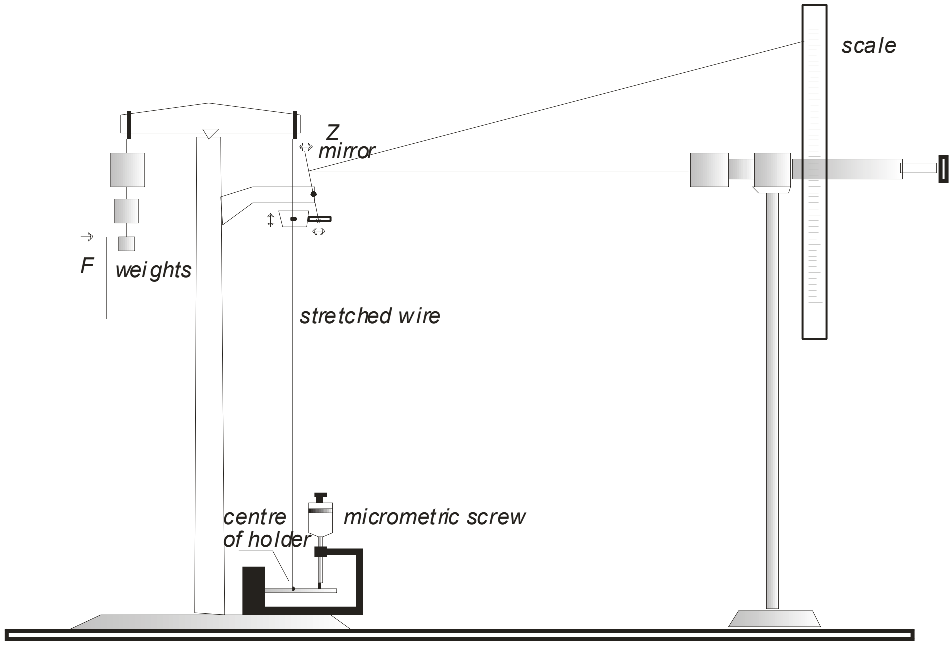

Fig.3

The steel test wire is supported between two ends of the laboratory stand. The force is applied to upper end of wire. The micrometric screw connects the bottom end of the wire. Because the elongation of the wire due to application of a force is very small, this it will be measured by the optical method. A knife-edge and carriers a small vertical mirror from which a spot of light is reflected support the upper end of wire. The reflected light fall on the screen that contains the scale. We can determine the change in the position of the spot on the scale. If we hang the various kinds of weights on the weigh pan we find that the elongation of the wire is to be proportional to the applied force. To calculation the Young modulus we must determine the relation between the change of the position of spot and the elongation of the wire. This relation is called the constant of device and it is defined as

$$k = \frac{h}{2y} \tag{6}$$

where h is the change in length of the wire due to the elongation by the micrometric screw (obviously 0.1mm) and y is the corresponding change in position of the spot on the scale. Inserting eq.6 into eq.2 gives the expression for determination of tensile strain as

$$\varepsilon = \frac{\Delta l}{l_0} = k\frac{\Delta y}{l_0} = \frac{h}{2y}\frac{\Delta y}{l_0} \tag{8}$$

where $\Delta y = y - y_0$ is the changed position of the light's spot after application the force mg. Note that y_0 is the position of spot without the external force mg. Therefore; the expression for the determination of the Young modulus is in form

$$Y = k\frac{l_0}{A}\frac{mg}{\Delta y} \quad (9)$$

MEASUREMENT

APPARATUS: tensile-strength apparatus, weights, source of light, micrometer screw, meter stick, telescope.

Measure the diameter of the wire with the micrometric screw a few times. Measure length of the steel wire l_0 between chucks. Determine the zero-weight position y_0. Measure the constant of device *k* of the optical lever for several times and record them into table. Note the step of the shift of micrometric screw is recommended as 0,1mm. Hang the mass of 0,5 kg at the top of wire. Record the change of the spot. Repeat the measurement for various values of weights and record them into table.

CALCULATION

Calculate the cross-sectional area *A* of the steel wire for any diameter as $A = \frac{\pi d^4}{4}$ and compute the average value of *A*. Using eq.6 calculate the constant *k* of device for each set of output values. Compute the average value of *k*. Calculate the tensile strain of the steel wire for any external force by the help of the eq.8

$$\varepsilon_i = \bar{k}\frac{\Delta y_i}{l_0} \quad \text{where } \bar{k} = \frac{1}{n}\sum_{i=1}^{n} k_i \quad (10)$$

Using eq.9 compute Young's modulus for each measured value as

$$E_i = \bar{k}\frac{l_0}{A}\frac{F_i}{\Delta y_i} \quad (11)$$

Calculate the average value of the Young modulus. Calculate the percentage error of measurement. The expression for the determination of the percentage error of measurement is in form

$$\frac{u_E}{E}100\% = \sqrt{\left(\frac{u_{l_0}}{l_o}\right)^2 + 4\left(\frac{u_d}{d}\right)^2 + \left(\frac{u_{m_i}}{m_i}\right)^2 + \left(\frac{u_k}{k}\right)^2} \cdot 100\% \quad (12)$$

where $u_{l_0}, u_d, u_{m_i}, u_k$ are the accuracies of the meter-stick, micrometric screw, weights and optical apparatus, respectively. Compare your result with accepted value of Young's modulus. Discuss the sources of errors in this experiment