9. ELECTRIC CIRCUITS

We shall now consider situations involving electric charges in motion. The term **electric current** is used to describe the rate of flow of charge through some region of space. To define current precisely, suppose the charges moving perpendicular to a surface area *A*, as is shown in Fig.9.1. The current is the rate at which charge flows through this surface. If ΔQ is the charge that passes through this area in a time interval Δt, **the average current**, I_{av}, is defined as

$$I_{av} = \frac{\Delta Q}{\Delta t} \ (\mathrm{C/s}) . \tag{9.1}$$

If the rate of charge flows varies in time, the current also varies in time and we defined **the instantaneous current**, *I*, as the differential limit of the eq.(9.1)

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} . \tag{9.2}$$

The SI unit of current is C/s which is called **ampere.** 1A of current is equivalent to 1C of charge passing through the surface in 1s.

When charges flow through the surface (see Fig.9.1), they can be positive, negative or both. It is conventional to choose the direction of the current to be in the direction of flow of positive charges. In a conductor such as copper, the direction of the current will be opposite the direction of flow of electrons. If we consider a beam of positively charge protons in accelerator, the current is in direction of motion protons. In some case, the current is the result of the flow of both positive and negative charges. This occurs, for example, in electrolytes or semiconductors.

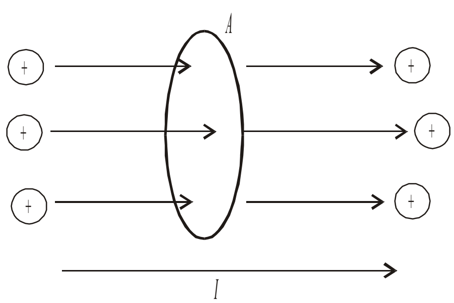

Fig.9. 2

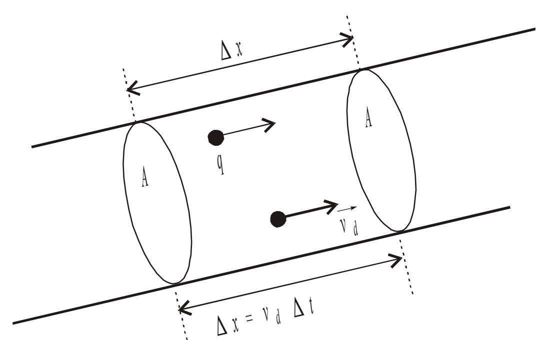

Fig.9. 1

It is instructive to relate the current to the motion of the charged particles. Consider the current in a conductor of cross-sectional area *A*, as is shown in Fig.9.2. The volume of an element of the conductor of length Δx is $A\mathrm{d}x$. If *n* represents the number of mobile charge carriers per unit volume, then the number of mobile charge carriers in this volume element is $nA\Delta x$. Therefore, the charge ΔQ in this element is given by

$$\Delta Q = (nAv_d \Delta t)q \, . \tag{9.3}$$

If we divide both sides of this equation by Δt, we have

$$I = \frac{\Delta Q}{\Delta t} = nAv_d q \, , \tag{9.4}$$

where v_d is actually an average velocity which is called **the drift velocity**. To understand this velocity, consider a conductor in which the charge carriers are free electrons. In isolated conductor, these electrons undergo random motion similar to the motion of gas molecules. When a potential difference is applied across the conductor, an electric field is set up in conductor, which creates an electric force on the electric charges (electrons) and hence a current. In reality, the electrons do not simply move in slight line along the conductor. They undergo collisions with the metal atoms, which result is complicated zigzag motion. The energy transferred from the electrons to the metal atoms causes an increase in the vibrational energy of the atoms and corresponding increase in the temperature of the conductors. However, despite the collisions, the electrons move slowly along the conductor with an average velocity called the drift velocity, v_d. The field does work on the electrons that exceeds the average loss due to collisions, which result is **a net current**. We see that **drift velocities are much smaller that the average speed between collisions**.

Example

Find the drift velocity of a copper wire of cross-sectional area $A = 3 \times 10^{-6}\ \mathrm{m}^2$ carriers of 10 A. The density of copper is $8.95 \times 10^3\ \mathrm{kg/m^3}$ and atomic weight of copper $M = 8.95\ \mathrm{g/mol}$.

Solution:

At first we calculate the volume V occupied by 8.95 g of copper as

$$V = \frac{m}{\rho} = \frac{63.5 \times 10^{-3}\ \mathrm{kg}}{8.95 \times 10^3\ \mathrm{kg.m^{-3}}} = 7.09 \times 10^{-6}\ \mathrm{m}^3 \, .$$

If we now assume that the copper atom contributes one free electron to the body of material, we can calculate the number of electrons in m^3 as

$$n = \frac{6.02 \times 10^{23}}{7.09\ \mathrm{m}^3} = 8.48 \times 10^{28}\ \mathrm{electrons/m^3}$$

Using eq.(9.4) we can determine the drift velocity as

$$v_d = \frac{I}{nqA} = \frac{10}{8.48 \times 10^{28} . 1.6 \times 10^{-19} . 3 \times 10^{-6}} = 2.46 \times 10^{-4}\ \text{m/s.}$$

We can see that the typical drift velocities are very small.

9.1 Ohm`s law

Charges move in a conductor to produce current under the action of an electric field inside the conductor. An electric field can exist in the conductor in this case since we are dealing with charges motion, **a nonelectrostatic situation**. This is in contrast with the situation in which a conductor in electrostatic equilibrium can have no electric field inside.

Consider a conductor of a cross-sectional area *A* carrying a current *I*. **The current density** j in the conductor is defined to be the current per unit area. Since $I = nqv_d A$, the current density is given by

$$j = \frac{I}{A} = nqv_d \quad (\text{A/m}^2) \tag{9.5}$$

This expression is valid only if the current density is uniform and the surface is perpendicular to the direction of the current. In general, the current is a vector quantity. That is

$$j = nqv_d \, . \tag{9.6}$$

A current density j and the electric field E are established in a conductor when a potential difference is maintained across a conductor. If the potential difference is constant, the current across the conductor will also be constant. Very often the current density in a conductor is proportional to the electric field in the conductor given by

$$j = \sigma E \, , \tag{9.7}$$

where the constant of proportionality is called **the conductivity** of the conductor. This equation is called **Ohm`s law** (1787-1854) and says that for many materials (including most metal) the ratio of the current density and electric field is constant, σ, which is independent of the electric field producing the current.

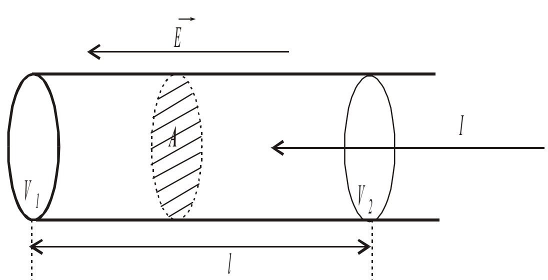

Fig.9. 3

For practical applications can be obtained another form of Ohm`s law by considering a segment of straight wire of cross-sectional area *A* and length *l* as is shown in Fig.9.3. A potential difference $V_2 - V_1$ is maintained across the wire, creating an electric field in the wire and current.

If the field in the wire is constant (uniform field), the potential difference is related to the electric field as

$$V_2 - V_1 = \int_1^2 \vec{E} \cdot \mathrm{d}\vec{r} = E\int_0^l \mathrm{d}x = El\ . \tag{9.8}$$

Therefore, we can expressed the magnitude of current in the wire as

$$j = \sigma E = \sigma \frac{V}{l}\ . \tag{9.9}$$

Since $j = I / A$, the potential difference can be written

$$V = \frac{l}{\sigma} j = \frac{l}{\sigma A} I\ . \tag{9.10}$$

The quantity $l / \sigma A$ is called **the resistance *R*** of the conductor

$$R = \frac{l}{\sigma A} = \frac{V}{I} \quad (\text{V/A}=\Omega). \tag{9.11}$$

From this expression we see that resistance has SI units of volts per ampere. One volt per ampere is defined to be one ohm (Ω).

The inverse of the conductivity of a material is called **the resistivity** ρ

$$\rho = \frac{1}{\sigma}\ . \tag{9.12}$$

Using eq.(9.11), the resistance can be expressed as

$$R = \rho \frac{l}{A}, \tag{9.13}$$

where ρ has the unit Ω.m. Every ohmic materials has a characteristic resistivity that depends on the properties of the material and on the temperature. For example, resistivity of copper is $1.8 \times 10^{-8}\ \Omega\mathrm{m}$ and iron is $1 \times 10^{-8}\ \Omega\mathrm{m}$. **Ohmic materials**, such a copper, have a linear current-voltage relationship over a large range of applied voltage. The slope of the *I* versus *V* curve in the linear region yields a value for *R*. **Nonohmic materials** have a nonlinear current-voltage relationship as is shown in Fig.9.4.

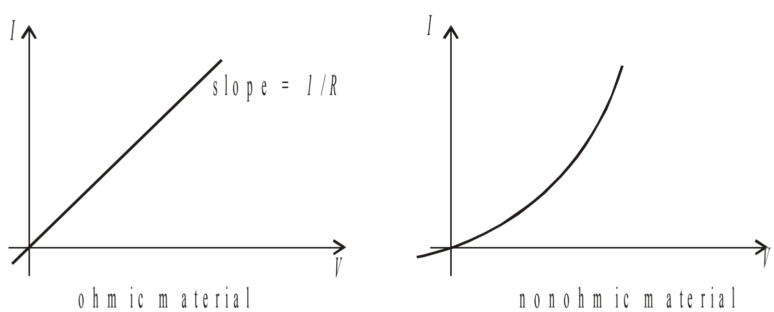

Fig.9. 4

The resistivity of a conductor depends of a number of factors, one of which is temperature. For most metals, resistivity increases with increasing temperature in an approximately linear fashion according to the expression

$$\rho = \rho_0 \left[1 + \alpha (t - t_0) \right], \tag{9.14}$$

where ρ is resistivity of some temperature in ˚C, ρ_0 is the resistivity at some reference temperature t_0 (usually taken to be 20˚C) and α is called **the temperature coefficient of resistivity**. From eq.(9.14) we see that

$$\alpha = \frac{1}{\rho_0} \frac{\Delta \rho}{\Delta t} \ (1/°\mathrm{C}). \tag{9.15}$$

Equation (9.15) is the definition of the temperature coefficient of resistivity where $\Delta \rho = \rho - \rho_0$ is the change of resistivity in a temperature interval $\Delta t = t - t_0$. For example, the temperature coefficient of resistivity of copper is $3.9 \times 10^{-3}\ (°\mathrm{C})^{-1}$.

Since the resistance of a conductor is proportional to the resistivity according to equation (9.13), the temperature variation of the resistance can be

$$R = R_0 \left[1 + \alpha (t - t_0) \right]. \tag{9.16}$$

9.2 Electrical structure of matter

We know that the basic constituens of all atoms of matter are charged particles. For example, when a metalic filement is heated, it emits electrons, just as molecules are vaporized when a liquid is heated. This phenomenon is called **thermionic emission**.

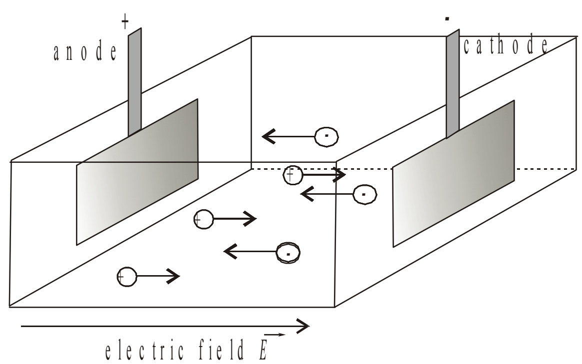

Fig.9. 5

Another interesting phenomenon is **the electrolysis**. Let us suppose that an electric field E is produced in molten salt (KHF_2) or a salt (NaCl) as is shown in Fig.9.5. The field is produced by immersing in the solution two oppositely plates charged called **electrodes**. Then we observed that electric charges flow so that the negative charged atoms move to positive electrode (anode) and the positive charged particle move to the negative electrode (cathode). This phenomenon suggests that the molecules of dissolved substance have separated (or dissociated) into two kind of charged parts, **ions**. For example, in case NaCl, Na atoms go to the cathode and therefore are positive ions, called **cations**, while the Cl atoms go to the anode and negative ions, called **anions**. The dissociation may be written in the form

$$\mathrm{NaCl} \rightarrow \mathrm{Na}^{+} + \mathrm{Cl}^{-}.$$

Each of the two parts is thus an ion. We have indicated that all charges are multiple of a fundamental unit charge *e*. Let us assume that the positive ions carry a charge $+\upsilon e$ and the negative ions a charge $-\upsilon e$, where υ is an integer to be determined later. When the ions arrive at each electrode, they become neutralized by exchanging their charge with the charge available at the electrodes. Usually there follows a series of chemical reactions that are of no concern to use here.

After a certain time *t* a number *N* of atoms of each kind has gone to each electrode. The total charge *Q* transferred at each electrode is then in absolute value, $Q = N\upsilon e$. If we assume that *m* is the mass of each molecule, the total mass *M* deposited at both electrodes is $M = nm$. Dividing the first relation by the second, we have

$$\frac{Q}{M} = \frac{\upsilon e}{m}. \qquad (9.17)$$

If N_A is Avogadro`s constant (the number of molecules in one mole of any substance), the mass of one mole of the substance is

$$M_A = nN_A. \qquad (9.18)$$

Therefore eq.(9.17) can be written

$$\frac{Q}{M} = \frac{\upsilon e}{m} = \frac{N_A \upsilon e}{N_A m} = \frac{F\upsilon}{M_A}. \qquad (9.19)$$

The quantity $F = eN_A$ is a universal constant called **the Faraday constant**. It represents the charge of one mole of ions having $\upsilon = 1$. Its experimental value is $F = 9.6487\times 10^{4}\ \mathrm{C/mole}$. From this value and the value of *e*, we obtained for Avogadro`s constant $N_A = 6.0225\times 10^{23}\ /\mathrm{mole}$ in agreement with other calculation of this constant. Equation (9.19) has been verified experimentally, and it has been found that υ is equal to **the chemical valence** of the ion concerned. The fact that υ is the chemical valence suggests that two atoms, when they bind together to make a molecule, exchange the charge υe, one becoming a positive ion and other a negative ion. The electrical interaction between the two ions holds them together.

9.3 Power

Consider a simple curcuit consisting of a battery connected to a resistor R as is shown in Fig.9.6. The positive terminal of the battery is at the higher potential, while the negative terminal is at lower potential.

The rate at which the average ΔQ losses potential energy is going through the resistor is given by

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} V = IV , \tag{9.20}$$

where we used the expression $\Delta U = q\Delta V$.

As the charge moves from point *1* to *2* through the resistor, it losses this electrical potential energy as it undergoes collisions with atoms in the resistor, thereby producing thermal energy. Note that we neglect the resistance of the interconnecting wires there is no loss in energy for path *3-4* and *1-2*. When the charge returns to point *3*, it must have the same potential energy (zero-reference point of grounded) as it had at the start.

Since the rate of which the charge losses energy equals the power *P* lost in the resistor, we have

$$P = IV . \tag{9.21}$$

In this case the power is supplied to a resistor by a battery. However, eq.(9.21) can be used to determination of the power transferred to any device carrying a current *I* and having a potential difference between its terminals.

Using eq.(9.21) and Ohm`s law for a resistor, we can expressed the power in the form

$$P = I^2 R = \frac{V^2}{R} \text{(watt)}, \tag{9.22}$$

where *I* is in amperes, *V* in volts and *R* in ohms. The SI unit of power is the watt. The power lost as heat in a conductor of resistance *R* is called **joule heating**.

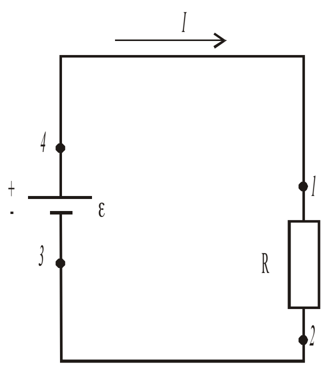

Fig.9. 6

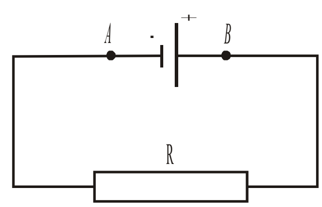

Fig.9. 7

9.4 Electromotive force

In order for a current to be maintained on a circuit, there must be device of curcuit element that is capable of continuously converting some form of energy into electrical energy. Such a device is called a **seat of electromotive force**. This force is abbreviated *emf*. A source of *emf* is any device that will increase the potential energy of charges circulating in a curcuit.

The seat of *emf*, ε , will take a charge dq that is at the point *A* at the negative terminal of the *emf* and move it to the point *B* at the positive terminal. In order to this, the *emf* will not flow back to point *A* by way of the resistor *R*, waless point *B* is at the higher potential than point *A* (see Fig.9.7). If the amount of work done is $\mathrm{d}W$, then *emf* is given by

$$\varepsilon = \frac{\mathrm{d}W}{\mathrm{d}q} . \qquad (9.23)$$

This equation is taken to be the definition for *emf*, ε . If you can see the *emf*, ε , of a source describes the work done per unit charge. Unit of electromotive force is 1 V. In the case of battery, the energy results from an **exothermic chemical reaction** that occurs in the battery.

Consider a circuit shown in Fig.9.8. We shall assume that connecting wires have no resistance. The positive terminal of battery is at a higher potential that the negative one. If we were to neglect the internal resistance of the battery, the potential difference across the battery would equal to *emf* of the battery. However, because a real battery always has some internal resistance *r*, the terminal voltage is not equal to

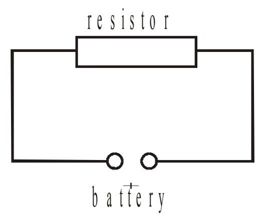

Fig.9. 8

the *emf* of the battery. Then curcuit shown in Fig.9.8 can be described by the curcuit in Fig.9.9. As the charge passes from the negative to the positive terminal of the battery, its potential increases by ε . However, as it moves through the resistance *r*, its potential decreases by an amount *Ir*, where *I* is the current in the circuit. Thus, the terminal voltage of the battery $V = V_2 - V_1$ is given by

$$V = \varepsilon - Ir . \qquad (9.24)$$

From this expression the terminal voltage equals ε , when the current is zero. In this case ε is equivalent to **the open-curcuit voltage**.

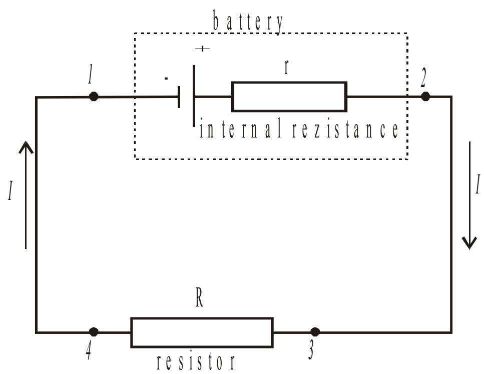

Fig.9. 10

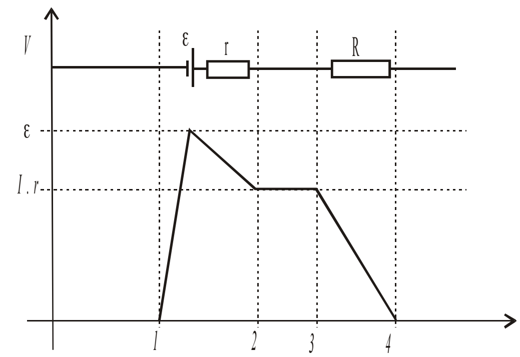

Fig.9. 9

Graphical representation of the changes in potential is in Fig.9.10. From this figure we see that the terminal voltage *V* must be equal the potential difference across the external resistance *R,* **called load resistance**. That is, $V = IR$. Inserting this value into eq.(9.24) gives

$$\varepsilon = IR + Ir \tag{9.25}$$

or

$$I = \frac{\varepsilon}{R + r}. \tag{9.26}$$

This equation shows that the current in the simple curcuit depends on both the resistance external to the battery and the internal resistance. If *R>> r*, we can neglect *r* in eq.(9.26). In many curcuits we shall neglect this internal resistance.

If we multiply eq(9.25) by the current *I* we have

$$I\varepsilon = I^2 R + I^2 r \tag{9.27}$$

This equation says that the total power output of the source of *emf*, $I\varepsilon$, is converted into power dissipated as joule heat in the load resistance, $I^2 R$, plus power dissipated in the internal resistance, $I^2 r$.

9.5 Kirchhoff`s rules

The procedure for analyzing more complex curcuits is greatly simplified by the use of two simple rules called Kirchhoff`s rules:

1. The sum of the current entering any junction must equal the sum of the currents leaving that junction. This rule is a statement of **conservation of charge**. That is, whatever current enters a given point in a curcuit must leave that point, since charge cannot build up at a point. If we apply this rule to the junction shown in Fig.9.11 we get

$$I = I_1 + I_2 \text{ or generally } \sum_k I_k = 0 \tag{9.28}$$

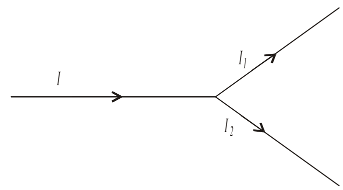

Fig.9. 11

2. This rule follows from **law of conservation of energy**. That is: any charge moves around any closed loop in a circuit must gain as much energy as it losses

$$\sum_k I_k \varepsilon_k = \sum_k I_k^2 R_k \quad (9.29)$$

or

$$\sum_k \varepsilon_k = \sum_k I_k R_k \,. \quad (9.30)$$

As we can applied the second rule, the following calculational tools should be noted:

a) If a resistor traversed in the direction of the current, the change in potential across the resistor is $-IR$ (Fig.9.12)

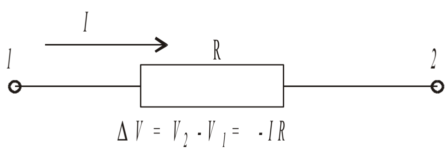

Fig.9. 12

b) If a resistor is traversed in the direction opposite the current, the change in potential across the resistor is $+IR$.

c) If a source of *emf* is traversed in the direction of the *emf* (from - to +) the change of potential is $+\varepsilon$, as is shown in Fig.9.13.

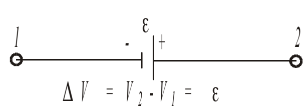

Fig.9. 13

d) If a source of *emf* is traversed in the direction opposite to *emf* (from + to -), the change in potential is $-\varepsilon$

Complex networks with many loops and junctions generate large number of independent, linear equations and a corresponding large number of unknowns. From these equations we can calculate the unknowns by neither matrix algebra or computers programs.