8. MEASUREMENT OF THE SHEAR MODULUS OF THE MATERIALS BY THE STATIC METHOD

ASSIGNMENT

1. Measure the shear modulus of various materials
2. Calculate the error of the measurement
3. Analyze the source of errors

THEORETICAL PART

Every substance is elastic to some degree. A material is to be elastic after a deformation of any kind, it returns readily to its original shape. There is a limit to the force, which may to applied to a body and still it return to its original, distorted state. This is called **the elastic limit**. If the stretching force is increased beyond the elastic limit, the material will eventually pull apart or break.

We shall discuss the elastic properties of solids in terms of the concept of **stress** and **strain**. We assume that a body is subjected to a force T tangential to one of it faces while the opposite face is held in fixed position (see Fig.14.1).

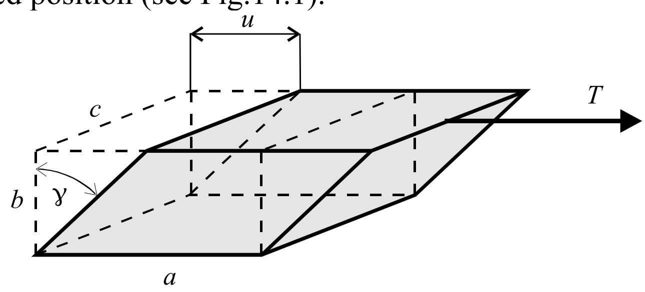

Fig.14.1

The object is originally a rectangular block, and the stress result in a shape, whose cross-section is a parallelogram. For this situation the stress called a **shear stress** is defined as

$$\tau = \frac{T}{ac} \tag{14.1}$$

and **the shear strain** as

$$\gamma = \frac{u}{b}. \tag{14.2}$$

From Hook's law follows the relation between the shear stress and shear strain in form

$$\frac{u}{b} = k\frac{T}{ac} \tag{14.3}$$

where the constant of proportionality is called the shear coefficient of elasticity. This constant is indirectly proportional to the modulus of elasticity in shear, *S*.
Therefore,

$$\tau = S\gamma \tag{14.4}$$

The unit of the modulus of elasticity in shear is Pascal (Pa).

Values of the shear modulus, for example, for steel are $8.4 \cdot 10^{10}\ \mathrm{Nm}^{-2}$ and for cooper are $4.2 \cdot 10^{10}\ \mathrm{Nm}^{-2}$ in contradistinction to the elastic modulus that are for steel $2\text{x}10^{11}\,\mathrm{Nm}^{-2}$ and for the copper $1\text{x}10^{11}\ \mathrm{Nm}^{-2}$.

Now we calculate the torque of torsion force acting on the uniform solid cylinder. The cylinder of radius *R* and length *l* is clamped at one end and the second end we will action with torque τ as is shown in Fig 14.2.

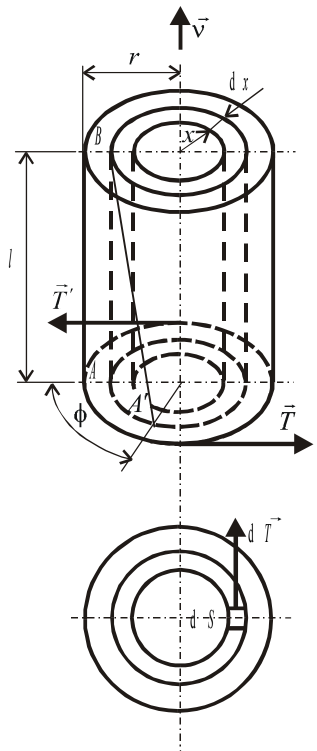

Fig 14.2

Assume that the upper end of the cylinder is attached and the second end is free to the swiveling. If the second end is swiveled about the angle Φ due to the torque τ that is parallel to the axis of the cylinder then the shells of the cylinder slide off. The relative slide of the cylinder is defined as

$$\gamma = \frac{\Phi x}{l} \qquad (14.5)$$

The magnitudes of all shells lying between the *x* and *dx* are the same. Then the shear stress acting on the lower base of the cylinder is

$$\tau = S\gamma = \frac{S\Phi x}{l}$$

This strain is the same in the element of the shell of the radius *x* and width *dx.* Its area is $2\pi x dx$. Therefore the tangential force acting on this area has the value

$$dT = \tau dA = \frac{S\Phi x}{l} 2\pi x dx$$

The torque according to the definition equals

$$d\tau = xdT = \frac{S\Phi x^3}{l} 2\pi dx$$

Therefore, the resultant torque on the base of the cylinder is

$$\tau = \int d\tau = \int_0^r \frac{S\Phi x^3}{l} 2\pi dx = \frac{\pi S r^4}{2l}\Phi \qquad (14.6)$$

From the expression follows that the angle Φ is proportional to the shear stress

$$\tau = \tau_0 \Phi \qquad (14.7)$$

$$\tau_0 = \frac{\pi S r^4}{2l} \qquad (14.8)$$

τ_0 is called turning moment of the torsion. It expresses the torque needed to the twisting of the bar about the unit angle.

THE METHOD - PRACTICAL PART

Eq. (14.6) represents a possible experimental technique for measuring the shear modulus of the materials. τ depends linearly on the angle Φ as is shown in eq.14.7. If we measure τ versus the angle Φ we can determine the shear modulus of the materials by the method of linear regression

MEASUREMENT

APPARATUS: The apparatus shown in Fig.14.3, two bars of various materials, weights, micrometric crew, slide caliper, linear scale

Measurement of the shear modulus is provided on the apparatus shown in Fig.14.3.

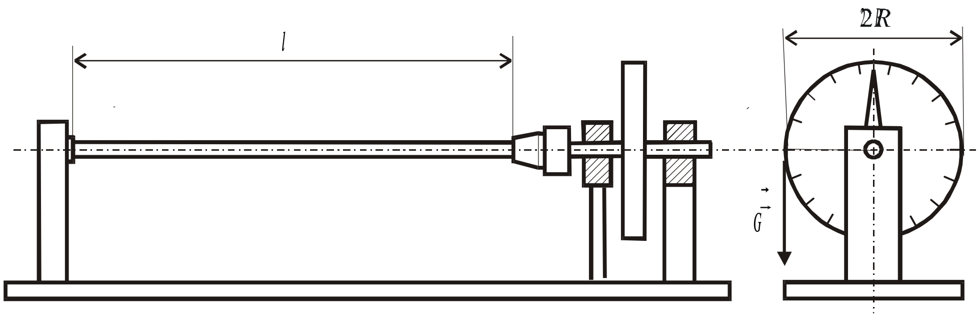

Fig.14.3

One end of the measured bar is fixed and the second end is attached in swiveling jaw connected with the circular disc. In the groove of the disc is attached to the stranded wire, which is loaded with the weight of mass *m*. The torque is by the definition equal $\tau = Rmg$. Vertex of the free end of the bar Φ will be measured on the angle scale of the disc.
Apply gradual the loads with the mass of 0.5 kg to the stranded wire and measure the angular displacement Φ for each applied load. Measure the diameter of the bar a few times carefully. Measure the length of the bar *l*.

CALCULATION

Calculate the shear modulus with the linear regression: General form of the linear equation is in form $y = a + bx$. For our case we have

$\tau = a + b\Phi$

where

$\tau_0 = b$, Φ is the angular displacement of each applied load and $\tau = mgr$

Using eq.14.8 the shear modulus may be calculated as

$$S = \frac{32kl}{\pi d^4} \qquad (14.10)$$

where *l* is the length of a bar and *d* is its diameter.

Note the parameter *a* must to be equal zero because it corresponds to the zeros value of the weight.

Using equation 14.11 calculate the error of measurement

$$\frac{u_S}{S} = \sqrt{\left(\frac{u_k}{k}\right)^2 + \left(\frac{u_l}{l}\right)^2 + 16\left(\frac{u_d}{d}\right)^2} \qquad (14.11)$$

Where

the value of u_k is given from calculation of the constant *k* by the linear regression, $u_l = \frac{1mm}{\sqrt{3}}$ and $u_d = \frac{0.001mm}{\sqrt{3}}$

Analyze the source of errors in the experiment