4. MEASUREMENT OF THE COEFFICIENT OF VISCOSITY OF THE LIQUID BY THE STOKES METHOD

OBJECTIVES

1. Measure the coefficient of viscosity
2. Analyse the errors in the measurement

THEORETICAL PART

The liquid is the state of matter cannot keep a fixed shape due to the molecular forces they are not strong enough to keep the molecules in fixed positions, and the molecules wander through the liquid in a random motion. When one tries to compress a liquid, strong repulsive forces act internally to resist the deformation.

Fluids have a certain amount of internal friction called **viscosity**. It is essentially a frictional force between different layers of liquid as they move. If the liquid layer is in contact with the stationary plate it remains stationary. This layer retards the flow of the layer above it and it layer retards to flow of the next layer above, and so on. Therefore, the velocity varies linearly from zero to value *v* as is shown in Fig.1

The increase in velocity divided by the distance over which this change is made equals

$$\frac{dv}{dy}$$

and is called **the velocity gradient**. The velocity gradient is defined as the rate the velocity changes per unit distance measured perpendicular to the direction of the velocity. To move the upper plate requires a force F. This force is proportional to the area A of the plate and to the velocity gradient as

$$F = \eta A \frac{dv}{dy} \qquad (1)$$

The proportionality constant η is called **the coefficient of viscosity of the liquid.** The SI unit of this coefficient is Pa.s. Its value depends on the kind of liquid and it is the function of the temperature. For example, the coefficient of viscosity of water at 20^0C is $1{,}0.10^{-3}$Pa.s and at the temperature 100^0C its value equals $0{,}3.10^{-3}$Pa.s.

THE METHOD - PRACTICAL PART

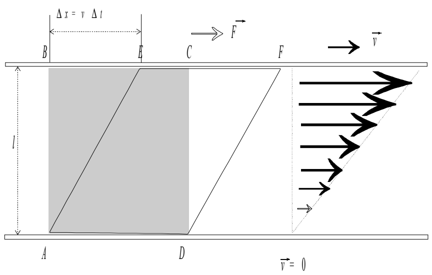

Fig.1

We shall measure the coefficient of viscosity of the liquid by Stokes' method. This method is based on the fact that the liquids offer resistance to the moving body. If the stream about the moving body is laminar, i.e. each particle of the liquid follows a smooth path, and these path don't cross over one other, the resistance of liquid is given by the Stokes' formula

$$F = - kv \tag{2}$$

where *v* is the velocity of moving body and *k* is the constant of proportionality. It depends on the geometry of the moving body and on the coefficient of viscosity. The negative sign means that the force has an opposite direction as is direction of the speed. For a small, solid sphere of mass *m* and radius *r* the resistance force is in form

$$F_s = - 6\pi r\eta v \tag{3}$$

Let us assume the small ball falling in liquid having the density ρ as is shown in Fig.2

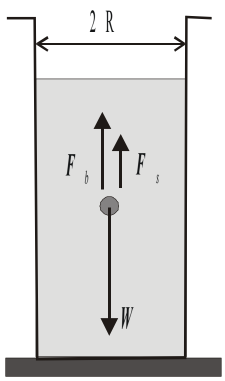

Fig.2

There three forces acting on the ball in liquid: The force of gravity W , the buoyant force F_b and the Stokes' force F_s .The ball moves in the liquid because of the force F equals the sum of all external forces, i.e.

$$\overline{F} = W + F_b + F_s \tag{4}$$

Archimedes' law gives the buoyant force. According to Archimedes' principle the buoyant force is equal to the weight of displaced liquid. It can be written

$$F_b = \rho V g$$

where: V is the volume of the liquid displaced by the ball

ρ represents the liquid density

g is acceleration of gravity

If you can see from Fig.2 the forces F_b and F_s act vertically upward through the centre of gravity of displaced liquid while the force of gravity W is oriented downward. Then the net force acting on the ball falling in liquid equals

$$F = mg - \rho V g - 6\pi r v \eta \tag{5}$$

Using the Newton's second law we have

$$ma = mg - \rho V g - 6\pi r v \eta \tag{6}$$

We can see that the ball undergoes an acceleration a .Since the third term in eq.6 depends on the velocity of the ball v, during the certain time the Stokes force compensated the term $(mg - \rho V g)$ and the acceleration equals zero. From this follows the constant velocity of the ball, approximately. Therefore, eq.6 transforms into form

$$0 = mg - V\rho g - 6\pi r v \eta \tag{7}$$

From this formula it follows immediately that

$$\eta = \frac{(m - \rho V)}{6\pi r v_0} g \tag{8}$$

Where v_0 is the terminal velocity when the acceleration of ball is zero. We note that this formula is valid if the dimension of the ball is neglect compare to dimension of the cylinder filled with the liquid. It means that we can neglect the friction force between the moving ball and the walls of the cylinder. If the interaction between the walls and ball can not be neglect we must introduced experimentally determined the correction term $2.4\frac{r}{R}$ where r is the radius of the ball and R is the radius of the cylinder. Then eq.8 is in form

$$\eta = \frac{g(m - \rho V)}{6\pi r\left(1 + 2.4\frac{r}{R}\right)} \cdot \frac{t}{l} \tag{9}$$

since $v = \frac{l}{t}$, where l is the path of the ball if the speed is constant and t is the corresponding time measured. From eq.9 we can calculate the acceptable value of η if the ball is falling along the axis of the cylindrical vessel of radius R.

MEASUREMENT

APPARATUS: Calibrated cylinder, test liquid, hydrometer, thermometer, balls, laboratory balance, tweezers, meter stick, microscope

The mass M of ten balls determine by the laboratory balance. Before this measurement is made the balance should be zeroed. Measure the diameter d of each ball with the microscope and calculate the average volume of the ball. Determine the mass of ten balls on the laboratory balance and calculate the average value of the mass of the ball Measure the density ρ of the test liquid with the hydrometer and the temperature of liquid by the thermometer. Calculate the radius R of the calibrated cylinder from its measuring length. Measure the time t of the falling balls if its motion is uniform i.e. between two marks on the cylindrical vessel. Measure the length between these two marks.

CALCULATION

Compute the coefficient of the viscosity η from eq.9

$$\eta = \frac{g(m - \rho V)}{6\pi r\left(1 + 2.4\frac{r}{R}\right)} \cdot \frac{t}{l},$$

where g is the acceleration of gravity. Its value is $9{,}806\ \mathrm{m \cdot s^{-2}}$, m is the mass of the ball, r the average value of the radius of the balls, t is the average time of the falling balls and ρ is the density of tested liquid.

Calculate the average velocity of the falling balls and estimate the statistical error of this velocity.

Explain the source of errors in this experiment.