2. MEASUREMENT OF AN AREA OF IRREGULAR SHAPE BY THE POLAR PLANIMETER

OBJECTIVES

1. Measure a plane figure
2. Determine the errors of the measurement
3. Analyze the source of errors in your experiment

THEORETICAL PART

Let us assume that we have two functions $f_1(x)$ and $f_2(x)$ in the plane of *xy*, as is shown in fig.1. The area enclosed by these functions is given by

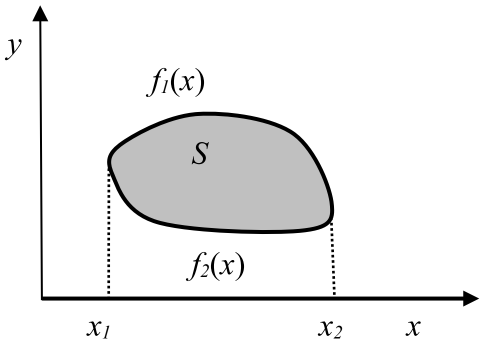

Fig. 1

$$S = \int_{x_1}^{x^2} f_1(x)dx - \int_{x_1}^{x_2} f_2(x)dx \qquad (1)$$

If the area of a graphically represented planar region has an irregular shape we can determine its magnitude by the measurement using the polar planimeter.

THE METHOD-PRACTICAL PART

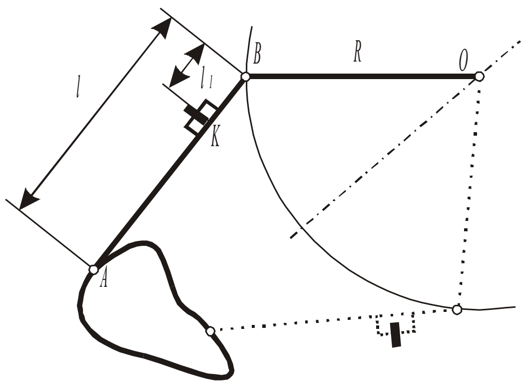

Fig.2

The polar planimeter is a drafting instrument used to measure the area of a graphically represented planer region, having any irregular shape. It consists of three parts: Anchor arm (polar arm) of the length R, traveling arm of length l and the linear with the point. The scheme of the polar planimeter is shown in fig.2.

The solid cylinder on the polar arm is anchored to the table with a point. It pivots, but does not slide. The elbow joint bents and slides freely. The pointer on the other end of the traveling arm is used to trace the perimeter of the region. Near the elbow of it is a wheel, which simply rolls and slides along the tabletop. On the wheel is the scale that tells how far the wheel has turned.

Let us assume that the pole of the planimeter is out of measured area, as is shown in Fig.3. If the end of the traveling arm (point A) is tracing the measured area S_a the second end of the arm (point B) rewrites the part of the circle in two opposite directions, i.e. the area S_b=0. Therefore, the magnitude of measured area S_a is proportional to the reading on the wheel, n,

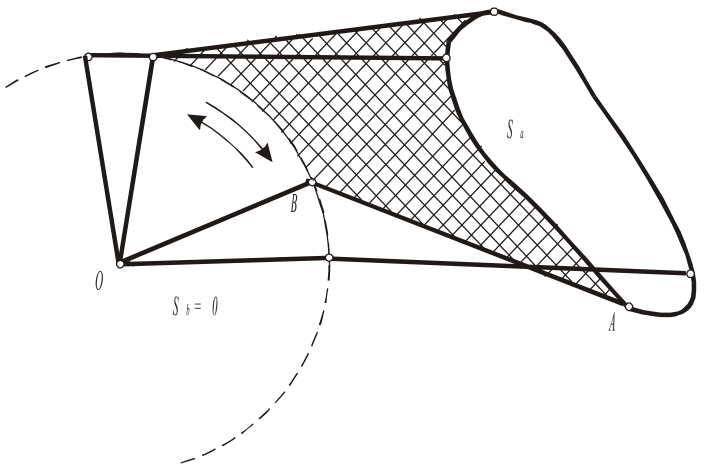

Fig.3

$$S_a = S = kn \qquad (2)$$

where k is called the conversion factor that depends on the scale of the drawing. This factor may be determined by the measuring the known area such as circle. If the area of the circle is S_0 and corresponding data on the wheel is n_0 then the value of conversion factor k determines the equation

$$k = \frac{S_0}{n_0} \qquad (3)$$

MEASUREMENT

APPARATUS: polar planimeter, linear, measured area

Measure the conversion factor: Establish the linear with a point on the table and free end of it set up to the traveling arm. (Point A). Before the measurement set up the wheel in "zeroed" position. Read this value in Tab.1.

i	n_{01}	n_{02}	n_{02}-n_{01}=$5n_0$	$\Delta 5n_0$	$(\Delta 5n_0)^2$
1.					
5.					
		Sum			
		$\overline{5n_0}$			
		$\overline{n_0}$			

Tab. 1

When the linear make one revolution around the circle the traveling arm also moves and we can read the data on the wheel due to one revolution of the linear. The area circumscribed, S_0, is introduced on the surface of the linear. Repeat the measurement nine times and read the measured values in the table. Measure a plane figure by the same manner and read the measured values in Tab.2.

i	n_1	n_2	n_2- n_1=$5n$	$\Delta 5n$	$(\Delta 5n)^2$
1.					
5.					
		Sum			
		$\overline{5n}$			
		$\overline{n}$			

Tab. 2

CALCULATION

Using eq.3 calculate the conversion factor. Using eq.2 calculate the plane area measured. Note that we have to insert the average values of n and n_0 calculated in Tabs.1 and 2.
From eq.4 calculate the standard error of measurement:

$$u_s = \sqrt{\left(\frac{\partial S}{\partial S_0}u_{S_0}\right)^2 + \left(\frac{\partial S}{\partial n_0}u_{n_0}\right)^2 + \left(\frac{\partial S}{\partial n}u_n\right)^2} = \sqrt{\left(\frac{n}{n_0}u_{S_0}\right)^2 + \left(\frac{S_0}{n_0}u_n\right)^2 + \left(-\frac{S_0 n}{n_0^2}u_{n_0}\right)^2} \qquad (4)$$

where

$$u_{n_0} = \frac{1}{5}\sqrt{\frac{\sum_{1}^{5}(\Delta 5 n_0)^2}{n(n-1)}} \qquad u_n = \sqrt{\frac{\sum_{1}^{5}(\Delta 5 n)^2}{n(n-1)}} \quad , \quad n = 5 \tag{5}$$

From eq.13.6 calculate the relative standard error

$$\frac{u_S}{S} = \sqrt{\left(\frac{u_{S_0}}{S_0}\right)^2 + \left(\frac{u_{n_0}}{n_0}\right)^2 + \left(\frac{u_n}{n}\right)^2} \ , \tag{6}$$

where

$$u_{S_0} = \frac{z_{max}}{\sqrt{3}}$$

z_{max} can be determined from the last number of the numerical value.

Analyze the source of the errors in your experiment.