DETERMINATION OF THE CORRECT MATING CYLINDRICAL TEETH FLANKS PROFILES WHEN THE PATH OF CONTACT IS GIVEN

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ABSTRACT

Design and classification of gearing comes almost exclusively from the so called "technological method". It means that for a known form of one wheel (mostly a rack tool), a correctly mating form of a tooth flank of a mating wheel is determined. This well known and simple method also positively describes the path of contact of the mating profiles. In this article the authors come out from given shape of path of contact by design and classification of cylindrical gearing and present a new method for determining the correct mating profiles with the application of stochastic lattice.

*Keywords**:** Cylindrical gearing, path of contact, correct mating profiles, stochastic lattice*

INTRODUCTION

The task to correctly design the mating profiles of plane gearing has already been long since accomplished practically. This is especially true for the so called technological approach to solving this problem, i.e. for determining the "appropriate" profile of the toothed wheel if the profile of the mating wheel is given (the shape of the tool is already known for example). Somewhat different is the case where the correctly mating profiles are defined or derived from the shape of the path of contact. This so called design approach is used very rarely, mainly because it is quite unprofitable and irrational for the design of common involute gearings. When the gearing requires special characteristics though, the use and also the design of non-involute gearing may be the only possible solution. Analytical methods of deriving the equations of profile curves are in this case only useable for a path of contact that is analytically defined, continuous and differentiable. For example, if the path of contact is defined by arcs of a circle according to Fig. 1, it is possible to show that the parametric equations would have the form (1) acc. to [2]:

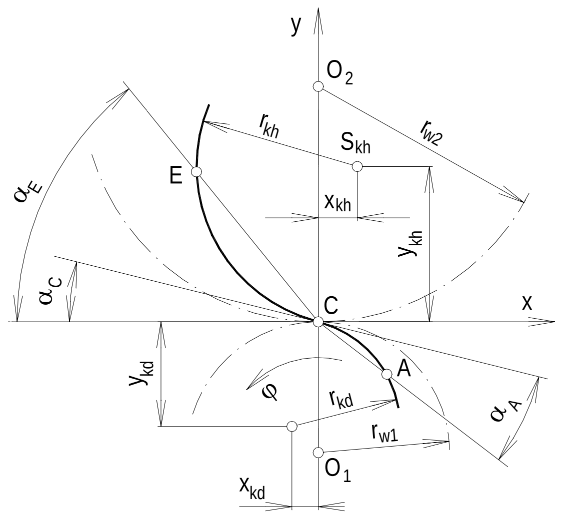

Fig. 1 Definition of gearing by path of contact.

$$x = \mp 2r_{kh,d}\sin(\alpha-\alpha_C)\cos(\alpha+\varphi_r(\alpha)) + r_1\sin\varphi_r(\alpha)$$
$$y = \pm 2r_{kh,d}\sin(\alpha-\alpha_C)\sin(\alpha+\varphi_r(\alpha)) + r_1\cos\varphi_r(\alpha) \tag{1}$$
$$\varphi_r = \pm\frac{2r_{kh,d}}{r_1}\left[(\alpha-\alpha_C)\cos\alpha_C + \sin\alpha_C \lg\frac{\cos\alpha_C}{\cos\alpha}\right]$$

Generally these equations define a convex-concave gearing. For some values of α_c and r_1 though, they also define a cycloid, pin and also involute gearing. If the path of contact is defined more generally, the analytical determination of the profile equations would be a lot more complicated and in some cases even impossible.

The need to solve this problem is very topical especially in common application programs for the design of toothed wheels which can be implemented in CAD systems. Here it is favorable if one doesn't have to work with restrictions for the shape of the path of contact, and if the whole process doesn't require special knowledge about the theory of gearing, while being universal and sufficiently exact. The original approach to solving this problem described further on, does have these required characteristics, since it solves this practical deterministic problem with probability distribution methods. It is also universal, providing a solution for common involute gearings as well.

STOCHASTIC LATTICE METHOD

Let $y = \varphi(x)$ be a continuous, piecewise smooth function, describing the curve of contact in an orthogonal coordinate system. Let R be a radius of the pitch circle. Let us assume the plane area Ω, such that the following conditions are satisfied:

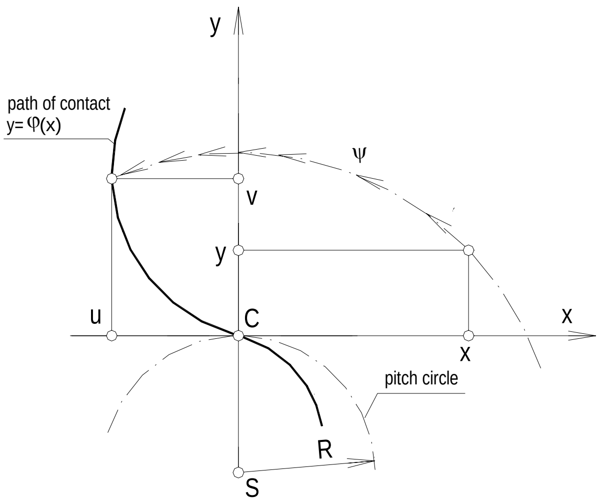

Fig. 2 Two dimensional transformation.

For each point $[x, y] \in \Omega$ there exist the uniquely determined point $[u, v] \in E_2$ such that:

$$v = \varphi(u) \tag{2}$$

$$x^2 + (R+y)^2 = u^2 + (R+v)^2 \tag{3}$$

Due to the above mentioned conditions it is possible to define the two-dimensional transformation

$$\Psi : \Omega \to E_2, [x, y] \to \Psi([x, y]) = [\xi(x, y), \eta(x, y)] = [u, v] \tag{4}$$

Typical representatives of such curves of contact are for example, the straight line corresponding to the involute type of gearing or the circle arches defining the cycloidal or convex-concave one. In the last mentioned situation it is possible to obtain values by means of using fundamental trigonometric methods. In a general case the use of appropriate numerical methods are required. The situation is demonstrated in a transparent symbolical way in Fig.2.

For the sake of simplicity, let us assume such part of the stochastic lattice where the case is an analogical one.

Let $[0,0] \xrightarrow{\pi_0} [\alpha_1, \beta_1] \xrightarrow{\pi_1} \ldots [\alpha_k, \beta_k] \xrightarrow{\pi_k} [\alpha_{k+1}, \beta_{k+1}] \xrightarrow{\pi_{k+1}} \ldots \xrightarrow{\pi_{n-1}} [\alpha_n, \beta_n]$

be a stochastic process on Ξ. Let us define the transition probabilities π_k in the following way:

If $[\alpha_k, \beta_k] = [x_i, y_j]$ then $[x_i, y_j] \xrightarrow{\pi_x(i,j)} [x_{i+1}, y_j]$, and $[x_i, y_j] \xrightarrow{\pi_y(i,j)} [x_i, y_{i+1}]$

where $\pi_x = \dfrac{\sin(\lambda_{ij})}{\sin(\lambda_{ij}) + \cos(\lambda_{ij})}$, $\pi_y = 1 - \pi_x$.

In the last formulas, the key role is played by λ_{ij} which is the angle between a touching line to the side profile of gearing in an actual gearing point and an abscissa joining the gearing point to the center of the voluble circle. The required values of goniometric functions can be obtained using the coordinates of the point $\Psi([x_i, y_j])$. The situation is shown in Fig.3.

An open polygon joining the neighboring points of each representation of the above determined stochastic process can be understood as a stochastic approximation of the side profile of gearing which corresponds to the curve of contact defined by the function dependency $y = \varphi(x)$.

Let $[0,0], [\alpha_1^k, \beta_1^k], [\alpha_2^k, \beta_2^k], \ldots, [\alpha_n^k, \beta_n^k]$ for $k = 1,2,\ldots m$ are m independent representations of this stochastic process. Due its definition, it is easy to see that an arbitrary point $[\alpha_q^k, \beta_q^k]$ can be represented merely by the couple $[x_i, y_j]$, for which $i + j = q$. For all fixed $q = 1,2,\ldots,n$ let us define the values

$$\Gamma_q(i) = card\{[x_i, y_{q-i}] : (\exists k \in \{1,2,\ldots,m\} : [x_i, y_{q-i}] = [\alpha_q^k, \beta_q^k])\}, i = 0,2,\ldots q \ .$$

Let us denote

$$\tau(q) = Me\{\Gamma_q(0), \Gamma_q(1), \Gamma_q(2), \ldots, \Gamma_q(q)\}$$

a median of the presented numerical set. An open polygon joining the neighboring points of a sequence

$$\{[0,0],[x_{\tau(1)},y_{1-\tau(1)}],[x_{\tau(2)},y_{2-\tau(2)}],[x_{\tau(3)},y_{3-\tau(3)}],\dots,[x_{\tau(n)},y_{n-\tau(n)}]\}$$

represents an approximate model of the side profile of gearing which corresponds to the above mentioned curve of contact. It is possible to achieve an arbitrary degree of precision, with reference to a number of realizations *m* and the grade of slightness of stochastic lattice determined by the constant Δ. The presented method is not sensitive to the complexity of the shape of curve of contact. The class of appropriate curves is an essential extension of the common generally known types. In order to demonstrate the efficiency of the approach presented here, let us show some concrete examples.

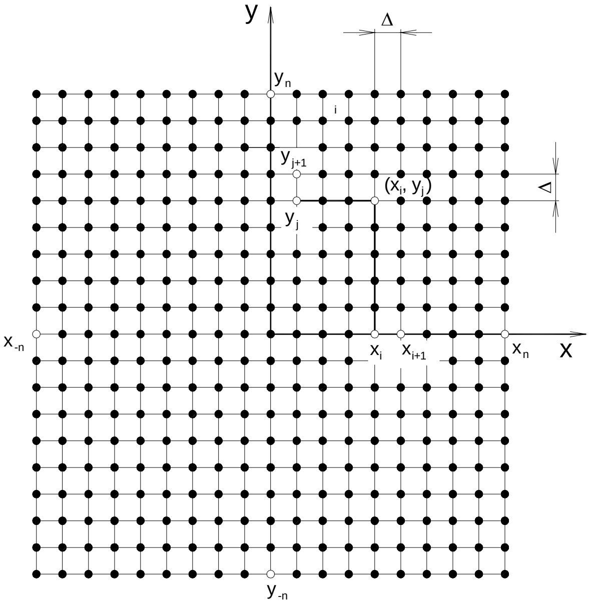

Fig. 3 Stochastic lattice.

The Figure 5 shows a classical case connected to the involute type of gearing. It is possible to see the contact line, a part of the voluble circle and the resulting shape of the flank profile of gearing obtained by use of the above mentioned method.

The “corridor” in the upper right hand corner of the picture represents the area in the exact centre of which would lay the analytically computed involute connected to the actual gearing. It is possible to see the strong coincidence between the obtained result and expected reality.

The situation shown in Figure 6. is principally different from the previous one. The curve of contact consists of two independent parts which have the following analytical representations:

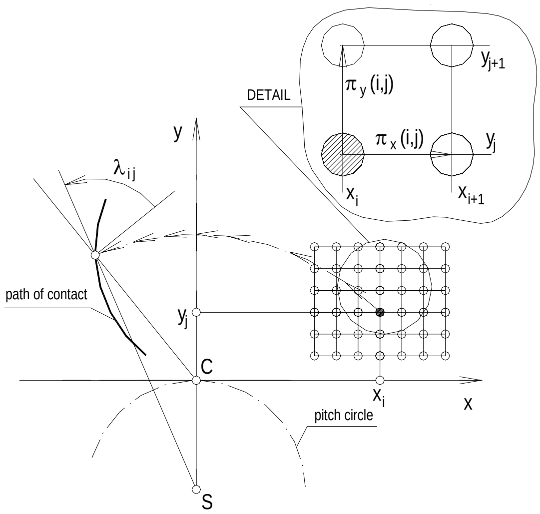

Fig. 4 Realization of stochastic process.

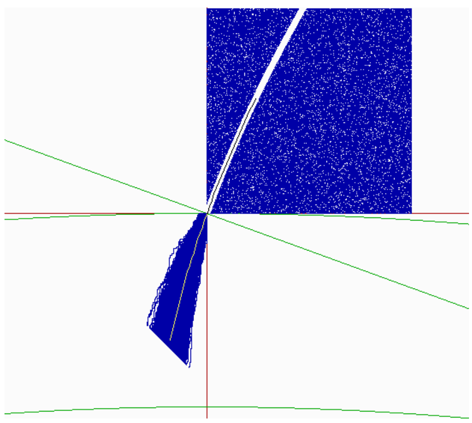
Fig. 5 Stochastic lattice method used for line type of path of contact (involute gearing)

- the upper (left) part : $\varphi(x) = 0.5\left(-0.01\,x^3 - 0.4x\right)$
- the low (right) part : $\varphi(x) = 0.5\left(-0.01\,x^2 - 0.4x\right)$ (5)

In both Figure 5 and Figure 6, we can see some results of individual stochastic processes which look like spidery ragged lines copying the used stochastic lattice. For the sake of completeness let us note that in the last example $\Delta = 0.04$, $n = 200$ and 2000 stochastic representations have been depicted. The possibility of using this procedure to create the rack form for a given shape of wheel gearing teeth is demonstrated in Figure 7a. Here was used the "interference gearing control" method. As stated previously, when the rack (basic rack) shape is given we can use the "technological method" to define the correct mating profiles for common geometrical parameters of arbitrary mating wheels.

The same procedure was used in the task to find the correct shape of teeth for correct mating gearing when the one of the coupling teeth shape is known (Fig. 8a, b).

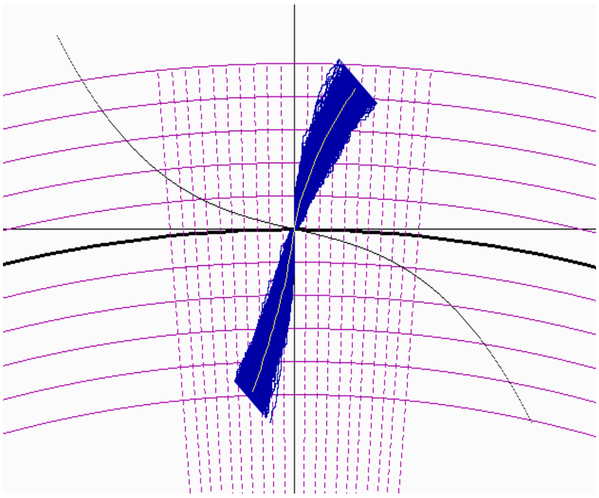

Fig. 6 Stochastic lattice method used for path of contact defined by two curves according. to expressions (5).

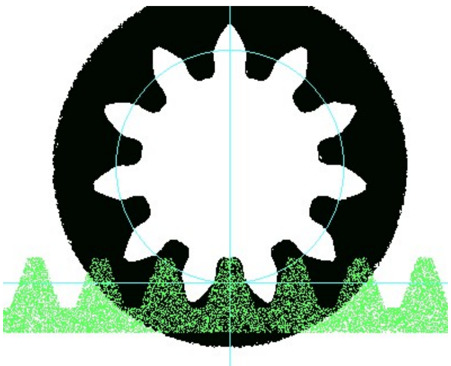

Fig. 7a Stochastic lattice method used to determine the rack teeth form when the wheel teeth shape are given.

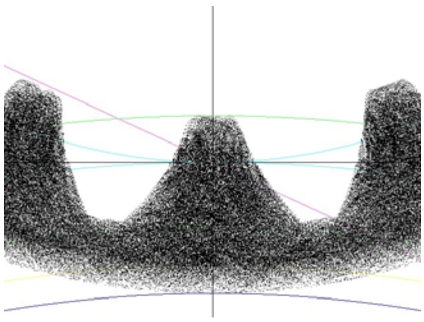

Fig. 7b Stochastic lattice method used to determine the teeth form of wheel when the mating pinion teeth shape is given.

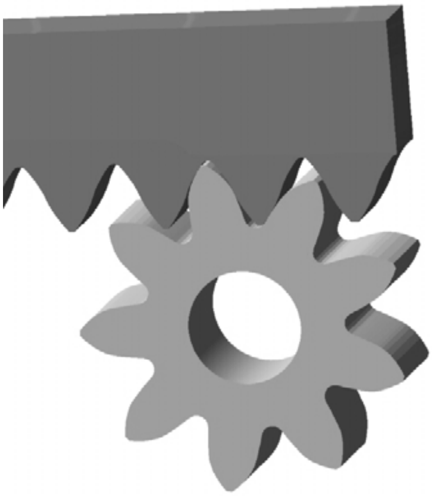

Fig. 8a 3D model of noninvolute gearing generated by CATIA system (as result of stochastic lattice method).

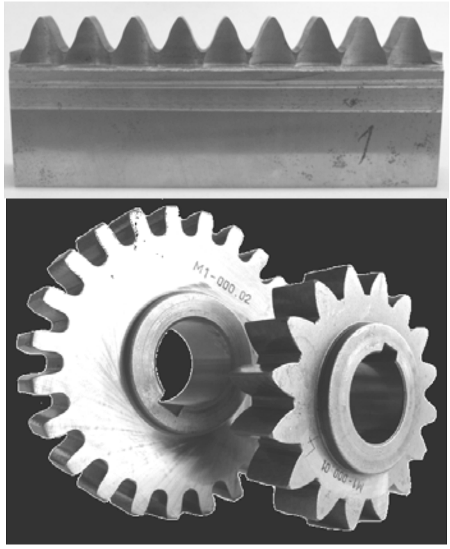

Fig. 8b Real rack and wheels designed and produced according the above described method.

CONCLUSION

The practical application of the described stochastic approach to the design of correctly fitting profiles of plane gearing was carried out in the CATIA v.5 system (Fig. 8, Fig. 9). The whole application procedure (macro) is quite simple and can be easily imported into other systems of computer aided design, or possibly even be used independently. The authors have also expanded the described procedure with interference of gearing control using the stochastic approach as well. This way it is possible to design operational gearing for basically any given shape of the path of contact.

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