SENSORS PLACEMENT FOR ACTIVE DAMPING OF VIBRATION ON TWO DIMENSIONAL PLATES

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ABSTRACT

This article deals with the investigation of optimal sensor placement for two dimensional systems with various boundary conditions. The approach proposed in this article is based on the evaluation of the H_2 and H_∞ norms. The optimal sensor and actuator placement satisfy the conditions of controllability and observability. The flexible structure is defined as a finite dimensional, controllable and observable linear system.
Keywords: Observability, sensor placement, norms, control

INTRODUCTION

To achieve the target of improving the performance control of flexible structures, it is useful to investigate various sensor and actuator locations. The purpose of the investigation is to determine the minimal number of sensors and actuators needed to meet the requirements of observability and controllability.

The importance of actuator and sensor placement is supported in many investigations and contributions. The articles [1] - [2] used the norms H_2, H_∞ and Hankel singular values for actuator and sensor placement. Contributions [3] - [4] use observability and controllability grammians for the actuator and sensor placement. The next big group of articles uses the various formulations of optimization problems [5] - [6] to find the solution to actuator and sensor placement in flexible structures.

NORMS

System norms serve as a measure of intensity of response to standard excitations, such as unit impulse or white noise of unit standard deviations. The standardized response allows the comparison of different systems. For flexible structures the H_2 norm has an additional property, which is a root-mean-square sum of the norms of individual modes. All norms of a mode with multiple inputs (outputs) can be decomposed into the rms sum of norms of a mode with a single input (output). Where (A, B, C) is a system state-space representation of a linear system, and let $G(\omega) = C(j\omega I-A)^{-1}$ be the transfer function.

The H_2 norm is formulated in form

$$\|G\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} tr(G^*(\omega)G(\omega)d\omega \qquad (1)$$

The H_∞ norm is formulated as

$$\|G\|_\infty = \sup \frac{\|y(t)\|_2}{\|u(t)\|_2} \tag{2}$$

where y(t) is the system output and u(t) is the system input.

SENSOR PLACEMENT

Placement of sensors and actuators is solved independently, and both procedures are similar. The transfer function is indicated by G, with all S candidate sensors. The index of placement σ2ki that evaluates the k-th sensor at the i-th mode in terms of the H_2 norm is defined with respect to all the modes and all admissible sensors

$$\sigma_{2ki} = w_{ki} \frac{\|G_{ki}\|_2}{\|G\|_2} \quad k = 1,...,S \; i = 1, ...,n \tag{3}$$

where wki ≥ 0 is the weight assigned to the k-th sensor and the i-th mode
n is the number of modes
Gki is the transfer function of the i-th mode and k-th sensor

The index of placement σ∞ki evaluates the k-th sensor at the i-th mode in terms of the H_∞ norm. This index is defined for all modes and all admissible sensors.

$$\sigma_{\infty ki} = w_{ki} \frac{\|G_{ki}\|_\infty}{\|G\|_\infty} \quad k = 1,...,S \; i = 1,...,n \tag{4}$$

The placement matrix gives an insight into the placement properties of each sensor because the index of placement of the k-th sensor is defined as the rms sum of the k-th column.
In case of the H_2 norm, it is the rms sum of the k-th sensor indices over all modes

$$\sigma_{sk} = \sqrt{\sum_{i=1}^{n} \sigma_{ik}^2} \quad k = 1 ,...,S \tag{5}$$

And in the case of H_∞ it is

$$\sigma_{sk} = \max_i(\sigma_{ik}) \quad i = 1,...,n \; k = 1, ...,S \tag{6}$$

EXAMPLE

Using H_∞ and H_2 norms for determining optimal sensor placement is presented in the following example dealing with a plane plate with boundary conditions at its both ends (Fig. 1). Calculation of natural frequencies and modes of the plate was done using finite element methods in program Ansys. The analyzed model of plate has six degrees of freedom in each node: displacements in directions x, y, z and rotations around these directions. The length of the plate is 50 centimetres, width is 40 centimetres and its thickness is 2,5 milimetres.
In the first case let us deal with a plate that is clamped on two sides. The modes of this plate are shown in Fig. 2.

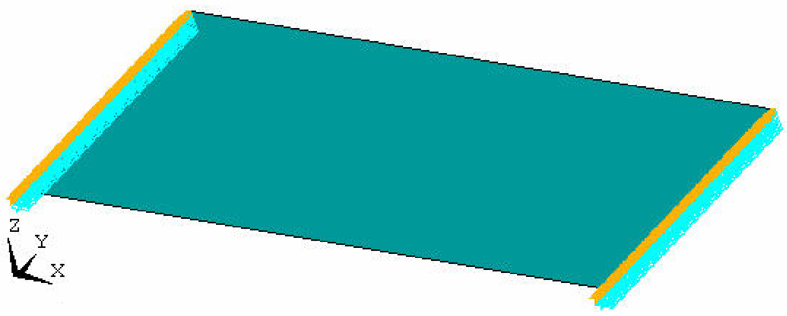

Fig. 1 Scheme of the plate with boundary conditions at the two shorter edges.

When an optical sensor for measuring an amplitude of vibration is used (Fig. 3), then it is needed to be placed perpendicularly to the surface of the plate for sensing the motion. Using the above presented H_∞ norm placement technique finds the best place for sensors functioning in z direction to control the first, second, third, and fourth own mode and to control simultaneously the first four modes.

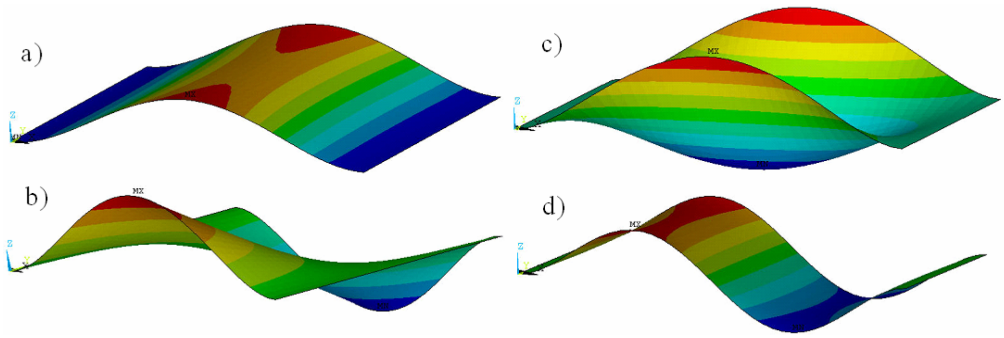

Fig. 2 Eigenmodes of the plate presentation. a) first mode, b) second mode, c) third mode, d) fourth mode

Fig. 3: Measuring of amplitude of vibration by optical sensor

We obtain H_∞ norm $\|Gki\|_\infty$ for the kth mode (k = 1, 2, 3, 4) and ith sensor location. From these norms we obtain the sensor placement indices for each mode from (4), using weight such that maxi ($\sigma\infty ki$) = 1. The plots of $\sigma\infty ki$ are shown in Fig. 3. The plot of the sensor placement indices for the first mode in Fig. 4a) shows the maximum in the middle of two longer edges, and indicates that a sensor should be placed at that position. In Fig. 4b) and 4c) indices for second mode and third mode reach their maximal values in the middle of two longer edges of the plate, as was the case of the first mode, although the second and third modes are different. Sensor placement indices for the fourth mode, reach two maxima on the both longer edges of the plate, which is shown in Fig. 4d).

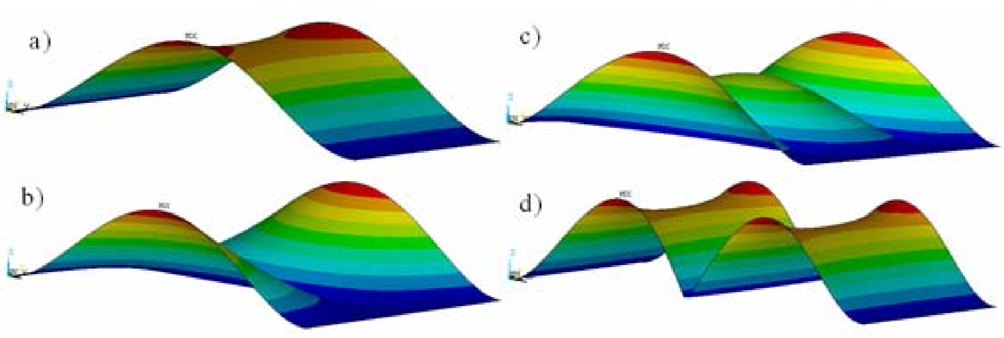

Fig. 4 Sensor placement indices as a function of sensor locations: a) for the first mode, b) for the second mode, c) for the third mode, d) for the fourth mode.

Next, we determine the indices for sensor placing for controlling the first four modes according to (6) σ∞1234i = max(σ∞1i,σ∞2i,σ∞3i,σ∞4i), we obtain three maxima on both the longer edges of the plate, which is shown in Fig. 5 a). For the first three modes we would obtain according to σ∞123i maximum from a position in the middle of the longer edge of the plate, as individually for the first mode and the second mode, and the third mode the index reaches its maximum at the same position.

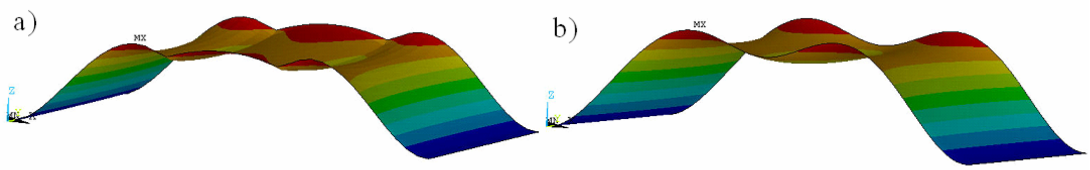

Fig. 5 Sensor placement H_∞ / H_2 indices as a function of sensor location.
a) for the first four modes of norm H_∞, b) for the first four modes of norm H_2

For determining the best actuator placement, the H_2 norm $\|G_{ki}\|_2$ for the kth mode (k = 1, 2, 3, 4) and ith actuator location was used for solving this example as well. From these norms and using (3) we obtain indices for each individual mode and from the equation (5) we determine the indices for the first four modes. Using $\sigma_{2,1234i}$ we obtain indices with two maxima at each longer edge, and these places are the most suitable for the sensor placement, shown in figure 5b).

Also piezoelectric sensors are suitable for use in active vibration suppression. Such a sensor with electrodes perpendicular to the axis of polarization P is shown in Fig. 6. With a piezoelectric sensor strain is measured on an outer layer during vibration.

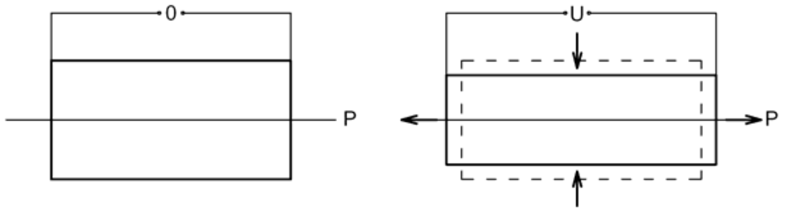

Fig. 6 Strain measurement by piezoelectric sensor.

In Fig. 7 we show the maximal strain during the first four single own modes of the plate clamped on the two shorter edges. These maximal strains are indices for placing sensors measuring a strain during vibration. As we can see sensor placement indices are maximal on the clamped edge but for every mode, in a different place. For measuring the strain with piezoelectric sensor for first four single modes, this sensor must be placed in a way that the axis of polarization is parallel to the longer edge.

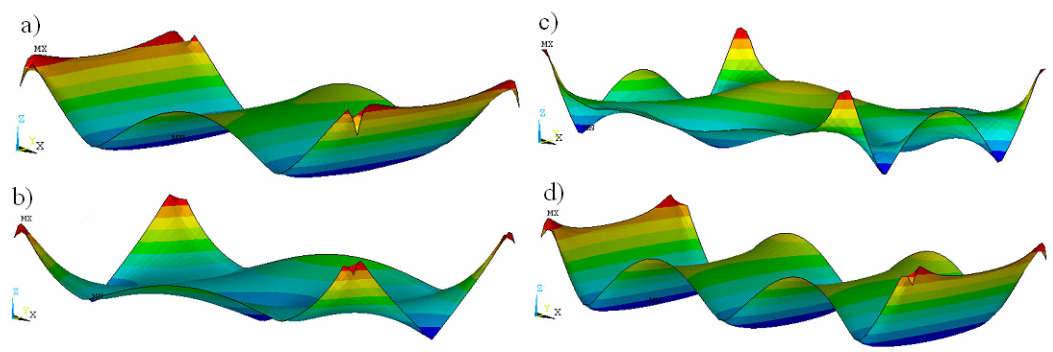

Fig. 7 Sensor placement indices as a function of sensor locations: a) for the first mode, b) for the second mode, c) for the third mode, d) for the fourth mode.

The H_∞ norm for the first four modes according to $\sigma\infty1234i = \max(\sigma\infty1i,\sigma\infty2i,\sigma\infty3i,\sigma\infty4i)$ shown in Fig. 8a) shows sixteen places altogether, eight places are on each shorter edge for sensor placing, where four are very close to each other. For the H_2 norm, using $\sigma_{2,1234i}$, there are indices with two maxima at each clamped edge, so four places are the most suitable for the sensor placement, which is shown in Fig. 8b).

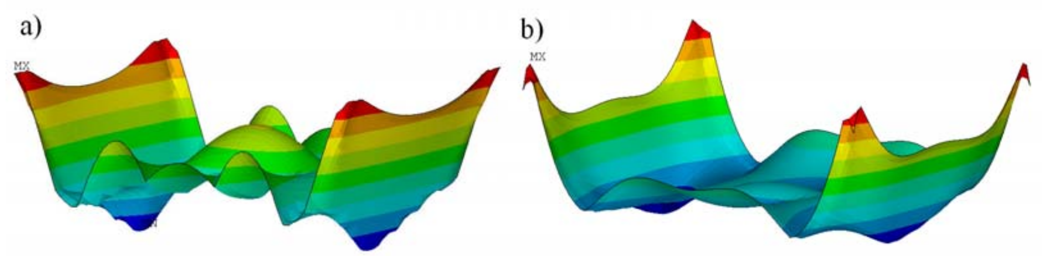

Fig. 8: sensor placement H_∞ / H_2 indices as a function of sensor location.
a) for the first four modes of norm H_∞, b) for the first four modes of norm H_2

CONCLUSION

When optical sensors for measuring the amplitude of vibration are used, then, according to norm H_∞ indices when the plate is clamped on shorter edges for the first four modes, there are six sensors on the plate. In the case of sensor placement according to indices of the norm H_2 for the first four modes, then we have only four sensors. For strain sensors there are different locations for optimal sensors placement. Clamped plate sensors should be placed next to the clamped edge. Following the evaluation of sensor placement according to norm H_∞ there are sixteen places for sensing, and for using norm H_2 there are four places for sensing.

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REFERENCES

[1] Gawronski W.K.: Advanced Structural Dynamics and Active Control of structures.Springer Verlag, New York, 2004.

[2] Liu,W., Hou,Z., Demetrion, M. A.: A computation scheme for the optimal sensor/actuator placement of flexible structures using spatial H2 measures. Mechanical Systems and Signal processing. Volume 20,4, 2006, pp.881-895.

[3] DeLorenzo M.L.: Sensor and actuator selection for large space structure control. Journal of Guidance, Control and Dynamics.Vol.13, 1990, pp.249-257.

[4] Kim Y., Junkins J.L.: Measure of controllability for actuator placement. Journal of Guidance, Control and Dynamics.Vol.14, 1991, pp.895-902.

[5] Gregory, Jr. C.Z.: Reduction of large flexible spacecraft models using integral balancing theory. Journal of Guidance, Control and Dynamics. Vol.7, 1984, pp.725-732.

[6] Yaowen,Y., Zhanli, J.,Chee,K.S.: Integrated optimal design of vibration control system for smart beams using genetic algorithms. Journal of Sound and Vibration 282(2005), 1293-1307.

[7] Šolek P., Horínek M.: Vyhodnotenie umiestnenia akčných členov v riadení, Applied mechanics 2009, Smolenice, 2009.

[8] Šolek P., Horínek M.: Optimal actuator / sensor placement for flexible structures, Acta Mechanica Slovaca, Košice, 2008.

[9] Halim D., Moheimani S. O. R.: Optimization approach to optimal placement of collocated piezoelectric actuators and sensors on thin plate. Mechatronics, 13:27-47, 2003.

[10] Šolek P., Horínek M.: Optimálne umiestnenie akčných členov v strojoch, ANSYS konference, Luhačovice, 2008.

[11] Šolek P., Horínek M.: Efectively placing sensors in machines and facilities, XII International conference on mechanical engineering, Bratislava, 2008.

[12] Šolek P., Horínek M.: Actuator/sensor placement for two-dimensional flexible systems, International conference MMAMS 2009, Zemplínska Šírava, 2009.

[13] Šolek P., Horínek M.: Active control of the two dimensional mechanical systems in resonance, International conference Mechatronics 2009, Luhačovice, 2009.

[14] Phillips D. J., Hyland D. C., Collins Jr. C. G.: Experimental demonstration of active vibration control for flexible structures. In IEEE Conference on Decision and Control, volume 4., pages 2024-2029, 1990.

[15] Gunei, Murat, Eskinat, Esref: Optimal actuator and sensor placement in flexible structures using closed-loop criteria. Jounal of Sound and Vibrations. Vol.312, 1-2, 2008, pp. 210-233.