NEW TIRE MODEL FOR TRANSMISSION OF THE FORCE IN VERTICAL DIRECTION

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ABSTRACT

This article is devoted to the issues of mathematical modelling of the dynamic behaviour of a tire in a vertical direction. The new tire model is specialized in the transmission of vertical force, in line with the results of the experiments carried out. After comparing the results of the experiment with the results of the available semi-physical tire models in MSC.ADAMS we found these models were not accurate enough. Consequently, we decided to develop a new tire model. For the new tire model it was necessary to design the methodology of the experiments, processing the data, evaluation of the dynamic characteristics of the tire and determine the mathematical model of the tire according to the results of the experiments. The final result of our work is the incorporation of the new model into MSC.ADAMS simulating software environment.

Keywords: Tire model, MSC.ADAMS, force transmission, dynamic stiffness, phase angle

INTRODUCTION

A mathematical model of the tire is a very important part of the full vehicle model. Forces and moments generated in the tire-road contact significantly influence the dynamic behavior of the whole car model. According to usage, tire models can be divided into those models used for simulation of driving maneuvers on the full vehicle model, and the structural tire models designated for special event simulations during driving maneuvers (e.g. overcoming obstacles). The following mathematical models can be distinguished:

a.) FEM tire models – detailed models for NHV, aquaplaning, wear analysis. These models are demanding on time and computational effort, and are not designated for full vehicle simulation.
b.) Physical tire models – mathematical model based on the physical principles, complex structure, nonlinear, very accurate – but still demanding of time and computational effort. They are designated for full vehicle simulation driving maneuvers. Famous models such as FTire, SWIFT-Tire, or RMOD-K belong to this group.
c.) Semi-physical tire models – mathematical models based on physical principles and results from experimentation. These models are computationally most effective but physical measurement of the real tire is necessary. such as Pacejka89, 94, 2002, MF-Tire, Fiala, UA-Tire, 5.2.1-Tire fall into this category.
In addition to the aforementioned well-known models, there are many models developed for specific purposes, just like our new model is.

FORCES IN THE TIRE

The complex mathematical model of the real tire, working under all types of the driving maneuvers, on all road types, with good computational efficiency, and with the full vehicle model is an enormous task. A real tire is an item of high complexity, comprising 150-200 types of materials (natural and synthetic elastomers, steel, nylon, carbon etc). All of these materials have different dynamic characteristics, so development of the computationally effective mathematical model reflecting properties of particular models is a demanding challenge.

During the contact between road and tire normal and friction forces are transmitted to all points of the contact surface. The effect of these forces is replaced by net force and moment vectors at the specific point on the contact surface. The computation of the forces and moments depends on the parameters of the tire (type, dimensions, pressure, stiffness of the tire, friction coefficient...), on the kinematical state of the wheel (longitudinal slip, slip angle, camber angle, rotational velocity ...) and on the parameters of the road (shape, friction coefficient, temperature...). This whole process is depicted in Fig. 1.

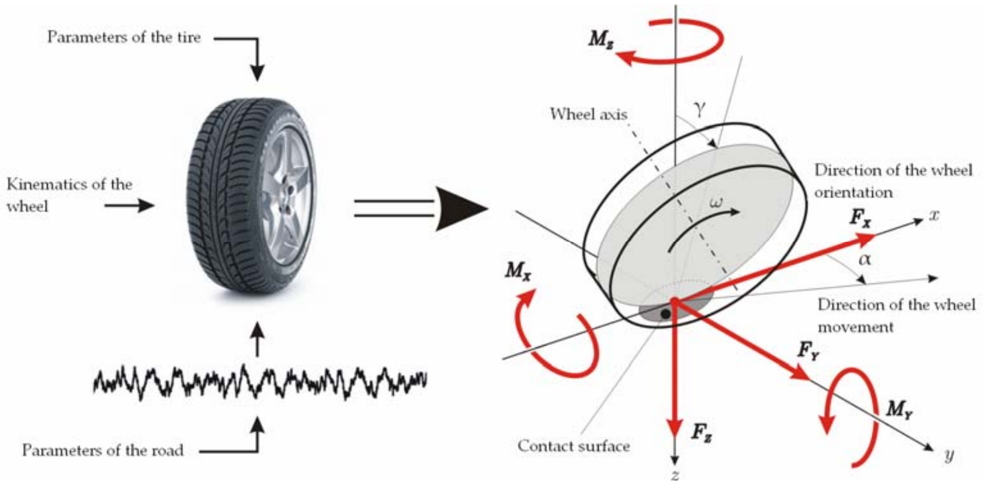

Fig. 1 Forces and moment in the contact point of the tire and road.

TIRE MODEL FOR TRANSMISSION OF TIRE FORCES IN A VERTICAL DIRECTION

After comparison the results of the experiment with the results of the simulations (produced by available semi-physical tire models in MSC.ADAMS) we found available semi-physical tire models not sufficiently accurate. This was the reason that we decided to develop a new tire model, specialized in vertical force transmission. This new model was developed in accordance with the results of the experiment.

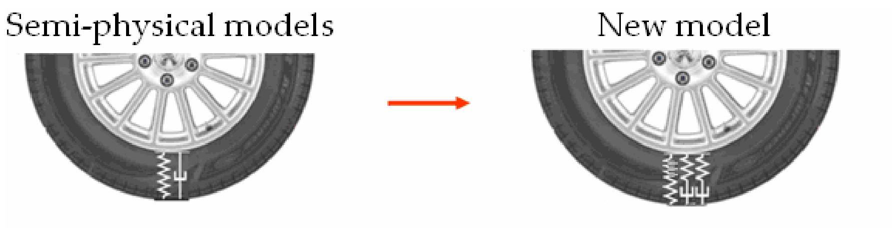

Fig. 2 Force elements of the tire models.

EXPERIMENT

From company Škoda Auto we obtained a set of measurements for the deformation and force response of the Bridgestone Turanza tire in the form of RPC III (*.rsp), which were done according to our instructions for sampling frequency, duration of record and positioning of the initial zero point. Together we obtained 18 records for 2 types of excitation:

a.) From 16 records of periodic excitation with 5 mm amplitude and frequencies of 0.01 Hz, 0.1 Hz, 1 Hz, 5 Hz, 10 Hz, 15 Hz, 20 Hz, for an inflation pressure of 250 kPa, and with 3 mm amplitude at 30 Hz frequency for an inflation pressure of 280 kPa.
b.) From stochastic excitation, 2 records at 30 Hz frequency for inflation pressures 250 kPa and 280 kPa.

The dynamic stiffness $Kdyn$ [N/m] and phase angle φ [rad] were chosen as the criteria for assessment of the suitability of the new tire model. These dynamic characteristics were evaluated from each record obtained during periodic excitation and finally were compared in frequency domain with stochastic courses.
The experiment is schematically drawn in the Fig. 3.

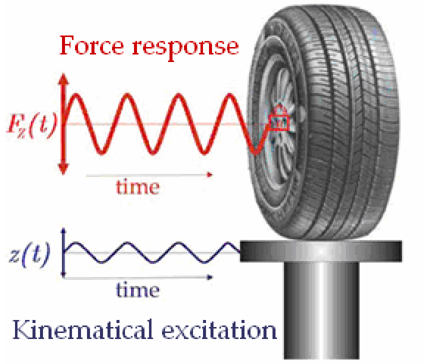

Fig. 3 Schematically depicted experiment.

DYNAMICS CHARACTERISTICS

The approximation of the excitation and the force behavior with Fourier trigonometric polynomial is in the general form:

$$FTP_{exc} = A_{0exc} + A_{exc} \cos\left(2\pi f t - \varphi_{exc}\right) \tag{1}$$

or

$$FTP_{force} = A_{0\,force} + A_{force} \cos\left(2\pi f t - \varphi_{force}\right) \tag{2}$$

where:
A_{0exc} and A_{0force} are the mean value of the excitation and force response,
A_{exc} and A_{force} are the amplitude of the excitation and force response,
φ_{exc} and φ_{force} are the phase shift of the excitation and force response.

From this approximation we can obtain the resulting dynamic characteristics – dynamics stiffness and phase angle.

Dynamic stiffness, as the ratio of the force response and excitation signal amplitude

$$Kdyn = \frac{A_{force}}{A_{exc}} \tag{3}$$

Phase angle, as the difference of force response and excitation signal phase angle

$$\varphi = \varphi_{force} - \varphi_{exc} \tag{4}$$

The advantage of the FTP approximation is its reliability even for the data with noise, but its disadvantage is the linearization of the approximated courses.

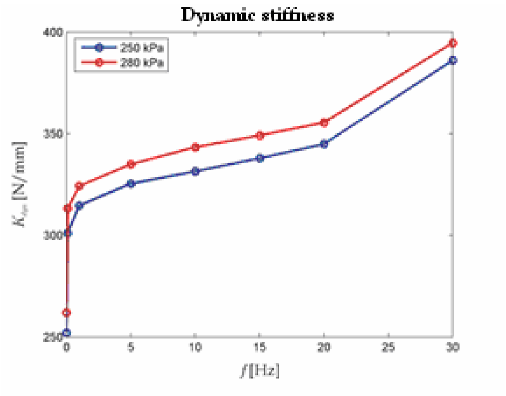

Fig. 4 Measured courses of the dynamic stiffness frequency domain.

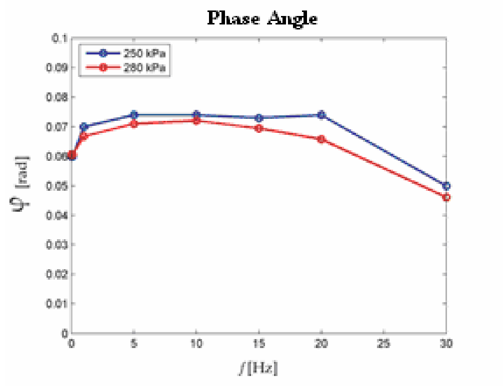

Fig. 5 Measured courses of the phase angle in frequency domain.

For optimization process it is more suitable to combine dynamic stiffness Kdyn and phase angle φ which change against frequency in a common graph in the form of complex dynamic stiffness $\bar{K}(i\omega)$ in the mathematical form

$$\bar{K}(i\omega) = Kdyn\, e^{i\varphi} \tag{5}$$

From this mathematical form we can conclude that the dynamic stiffness Kdyn is the absolute value and the phase angle φ is an argument of the complex dynamic stiffness $\bar{K}(i\omega)$ (Fig. 8)**.**

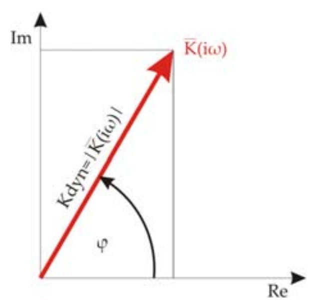

Fig. 8 Complex dynamic stiffness.

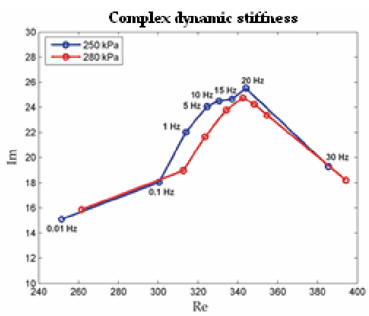

Fig. 9 Complex dynamic stiffness in Gauss plane

MATHEMATICAL MODEL

From a comparison of the results from the virtual test rig and from measurements, we detected that computationally effective standard tire models (FIALA, UA, Pacejka, MF) do not generate vertical forces with the desired accuracy. These models use the Kelvin-Voigt element for computing vertical force

From studying the different force elements (Maxwell, Kelvin-Voigt, Masing, Gehman) and their combinations, it was concluded that the best suitability for approximating the results of the measurement will have the element for force transmission, comprising the parallel combination of two Maxwell elements and a linear spring. The element was denoted as Maxwell2. Increasing the number of the Maxwell elements enables us to more precisely form the shape of the dynamic characteristics of the element. However, when the ADAMS/Tire element is used, it enables the user to define only two differential equations – two Maxwell elements.

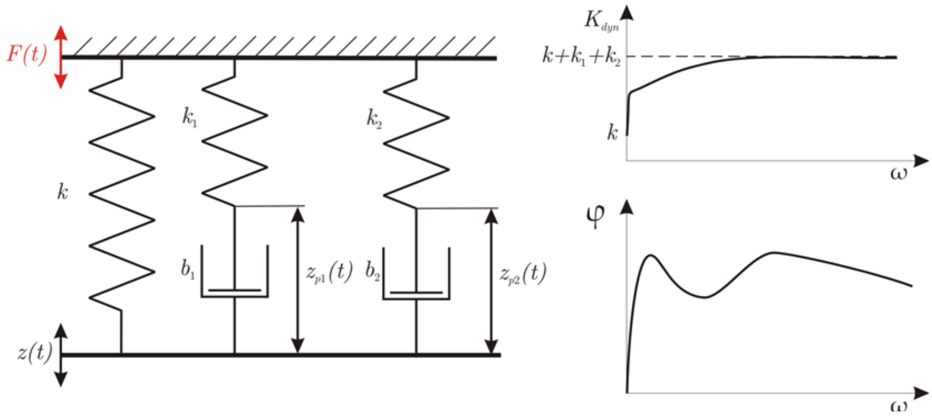

Fig. 10 Configuration in mechanic model of Maxwell2 force element.

The advantage of the Maxwell2 force element is that it enables us to achieve the desired shape of dynamic stiffness Kdyn diagram and simultaneously to maintain almost constant shape of phase angle φ diagram by appropriate adjustments of the stiffness k, k_1 and damping b, b_1 parameter values.

The explicit mathematical form of the Maxwell2 force element in the frequency domain is:

Complex dynamic stiffness $\bar{K}(i\omega)$

$$\bar{K}(i\omega) = k + \sum_{j=1}^{2} \frac{k_j \tau_j^2 \omega^2}{1+\tau_j^2 \omega^2} + i \sum_{j=1}^{2} \frac{b_j \omega}{1+\tau_j^2 \omega^2} \tag{6}$$

where $\omega = 2\pi f$ is angular frequency and $\tau_j = \frac{b_j}{k_j}$ is relaxation constant.

Dynamic stiffness $Kdyn$

$$Kdyn(\omega) = \sqrt{\left(k + \sum_{j=1}^{2} \frac{k_j \tau_j^2 \omega^2}{1+\tau_j^2\omega^2}\right)^2 + \left(\sum_{j=1}^{2} \frac{b_j \omega}{1+\tau_j^2\omega^2}\right)^2} \tag{7}$$

Phase angle φ

$$\varphi(\omega) = \arctan \frac{\sum_{j=1}^{2} \frac{b_j \omega}{1+\tau_j^2\omega^2}}{k + \sum_{j=1}^{2} \frac{k_j \tau_j^2 \omega^2}{1+\tau_j^2\omega^2}} \tag{8}$$

Explicit mathematical form of the force in Maxwell2 element in the time domain:

$$F(t) = kz(t) + b_1 \dot{z}_{p1}(t) + b_2 \dot{z}_{p2}(t) \tag{9}$$

where $\dot{z}_{p1}$ and $\dot{z}_{p2}$ are the internal variables defined by differential equations:

$$\dot{z}_{p1}(t) = \frac{k_1}{b_1}\left(z(t) - z_{p1}(t)\right), \quad \dot{z}_{p2}(t) = \frac{k_2}{b_2}\left(z(t) - z_{p2}(t)\right) \tag{10, 11}$$

We can see, that for calculation of the force response of the Maxwell2 element, it is necessary to solve two independent differential equations.

PARAMETER DETERMINATION

Due to the antagonistic properties of frequency dependent courses of dynamic stiffness $Kdyn$ and phase angle φ, the optimization of parameters was based on a strategy to achieve the best possible correlation of complex dynamic stiffness from computation and from measurements. The objective function was to minimise the value of $\min f$ (green difference in fig. 11) between the red complex dynamic stiffness from computation $\bar{K}(i\omega)_i^{VYP}$ and the blue from measurements $\bar{K}(i\omega)_i^{MER}$

$$\min f = \sqrt{\frac{1}{N-p-1}\sum_{k=1}^{N}\left|\bar{K}(i\omega)_k^{MER} - \bar{K}(i\omega)_k^{VYP}\right|^2} \qquad k = 1,2,\ldots,N \tag{12}$$

where N is number of measurements, $\bar{K}(i\omega)_k^{MER}$ is k-th measured complex dynamic stiffness and $\bar{K}(i\omega)_k^{VYP}$ is k-th complex dynamic stiffness computed from Maxwell2 force element.

For the purpose of the determination of the parameters k, k_1, b_1, k_2, b_2 of the Maxwell2 element was used the original stochastic climbing algorithm termed MNP (method of random search) [1].

This algorithm was used with the intention of investigating the usefulness of the theory of the n-dimensional hyper-sphere in the technical applications.
After successful optimization, we obtained values of the Maxwell2 parameters and subsequently compared them with measurements in the time and frequency domain. Figures 12, 13, 14 contain these comparisons plus diagrams generated by the Kelvin-Voigt element, used in all semi-physical tire models in MSC.ADAMS.

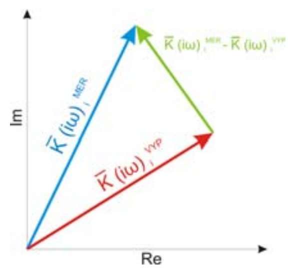

Fig. 11 Difference between measured and computed complex dynamic stiffness.

We can see that the Maxwell2 element generates characteristics in the frequency and time domains that are more suitable than the Kelvin-Voigt element in comparison with measurements. The quality of the correspondence of the measurement is quantified in [1].

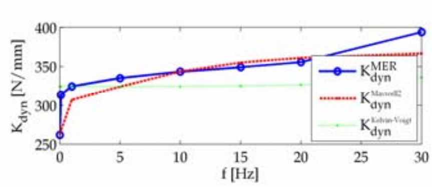

Fig. 12 Shapes of diagrams of the dynamic stiffness in the frequency domain.

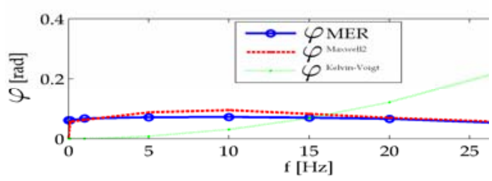

Fig. 13 Shapes of diagrams of the phase angle in the frequency domain.

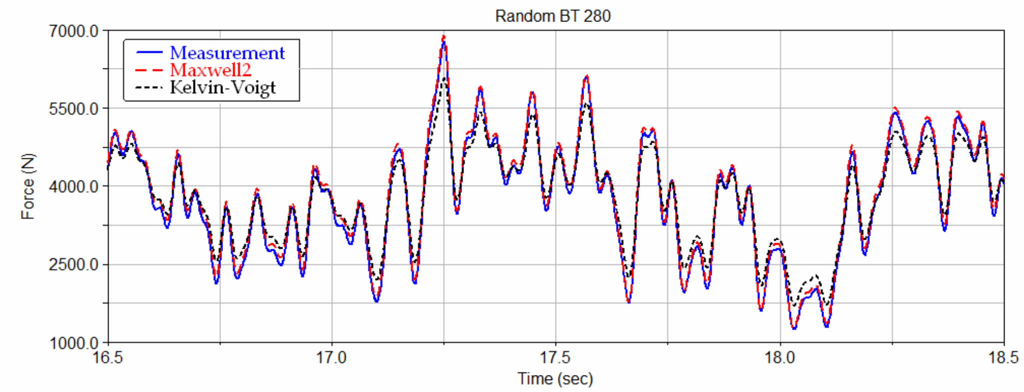

Fig. 14 Shapes of diagrams of the force responses in the time domain.

IMPLEMENTATION OF THE MODEL INTO MSC.ADAMS ENVIRONMENT

In order to use the new tire model in MSC.ADAMS is was necessary to adapt it to the program environment. We opted for the most complex way, the implementation via “Special force: Tire” on the basis of the FIALA model. This solution required the subroutine in FORTRAN and linkage of the dynamic library to the MSC.ADAMS. Although this solution was the most time consuming, it offers the best possibilities for the manipulation of the tire model. Users only have to input information about the tire model (the values of the coefficients of the Maxwell2 element, dimensions, friction) in the property file and link the dynamic library to the ADAMS/Solver. Since the new tire model is based on the FIALA tire model, it provides the information about the whole kinematical state of the wheel, and handling simulations are possible as well.

CONCLUSIONS

The goal of our work was to develop a new mathematical tire model specialized in the transmission of force in the vertical direction of the wheel suspension. The new tire model was designed using the results (dynamic stiffness and phase angle of real tire) from experiments. According to these characteristics the most appropriate force element was chosen– the so-called Maxwell2 element, whose dynamic characteristics allows the most suitable approximations of the dynamic characteristic of the real tire. After development of the mathematical model it was incorporated into the MSC.ADAMS simulation software and provided for Škoda Auto Mladá Boleslav.

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