SCIENTIFIC PROCEEDINGS 2009, Faculty of Mechanical Engineering, STU in Bratislava

MATHEMATICAL SIMULATION OF GEOTHERMAL HEAT TRANSFER IN HOT DRY ROCK UNDERGROUND HEAT EXCHANGERS

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ABSTRACT

The article describes a calculation method for non- steady state heat transfer in Hot Dry Rock massive. This method is used in geothermal applications focused on the exploitation of stored heat in deep hot earth underground which is quantitatively a much larger source of thermal energy than common geothermal underground waters. Obtaining of this kind of thermal energy requires the building of underground heat exchangers, drilled directly into rock massive. Heat carrying fluid which is usually river water or carbon dioxide pumped from the earth's surface in to the underground heat exchanger via drillings (wells) takes out the heat from the surrounding rock. Thus non-steady state heat transfer occurs in that surrounding rock massive which is continually sub cooled. On the other hand the same surrounding rock massive is continually heated from deeper layers of rock massive. The calculation of these heat transfer processes and experimental verification of this calculation is presented below.

Keywords: geothermal energy, heat transfer in hot dry rock, non-steady state heat transfer

INTRODUCTION

Geothermal energy belongs amongst the most promising alternative energy sources, especially Hot Dry Rock systems in which the heat is stored in a rock massive directly, and thus it is not bound to relatively scarce geothermal underground waters. Because quantitatively, this heat source is practically unlimited, it has a special importance for future energy supplies.

The reason that this heat source is not utilized industrially on a larger scale yet, is to be found, above all, in the relatively high costs for wells and in the uncertainties of making large volume underground heat exchangers (even one kilometre in diameter) which have to be created directly in underground rock massive. However the drilling technologies are progressing quickly and the first dozens of geothermal power plants based on Hot Dry Rock heat utilization are already functioning worldwide, including in the EU. Additionally, as Slovakia and the Carpathian region have generally particularly suitable geological conditions for such applications, the respective technologies, know how and design skills are becoming more and more important [1].

UNDERGROUND HEAT STORED IN ROCK

In general, the higher the temperature difference that is available in a thermodynamic cycle, the higher the quality (i.e. energy) the respective heat source has. Therefore, above all, heat sources with high temperature levels enable an effective exchange of heat into mechanical work. Low temperature heat sources under 130°C are not considered as suitable for energy transformation into mechanical work, and such a low potential heat is utilized directly and only for purposes such as district heating, or heating of greenhouses or swimming pools.

Therefore, if there is rock underground with temperatures above 150°C at reasonable depths (i.e. up to 4000 m), a geothermal power plant, based on the utilization of Hot Dry Rock heat becomes a real possibility. This is the case of some areas of Slovakia.

A Hot Dry Rock based geothermal system consists basically of a geothermal power plant based on the Organic Rankine cycle (ORC), coupled to injection and exploitation deep wells and underground heat exchangers [1].

There is no doubt that the most risky part of building such system is the underground heat exchanger.

One of the very important parts at the design of the above mentioned heat exchanger is the correct estimate of the necessary dimensions of heat exchanging surfaces between the hot rock and the heat carrying fluid, which is usually water.

The water (obviously river water) is pumped into an underground heat exchanger where it is heated by the surrounding hot rock. Therefore the rock massive is gradually cooled around the pipes of the exchanger, in which the heat carrying water flows. On the other hand, it is obvious,

that sub cooled rock parts are heated from the more distant, unaffected surrounding hot rock massive.

Thus, an interesting situation appears in the heat flows inside the underground rock heat exchanger. Principally, this thermo kinetic situation can be formally classified as a non steady state heat transfer in an unlimited massive or half limited massive.

CALCULATION OF HEAT TRANSFER

Logically, we need to know the heat transfer relations for determining the underground heat exchanger dimensions at required energy demands.

The determination of this key parameter i.e. heat transfer in the heat exchanger was carried out by following method:

We used the following simplifications in our modelling: we considered a pipe drilled into rock massive in which cold water flows as a geometric model. All calculations were carried out on this geometric model. The conductivity heat transfer coefficient α was calculated for 40°C water flowing turbulently through a pipe in 200°C rock massive. Granite was taken as the surrounding rock massive, with an average thermal conductivity of 2,5 W.(m. K)$^{-1}$. The physical properties of rock and water are indicated in Tab.1. and Tab 2.

Tab 1 Rock properties

Density	ρ_H	kg.m^{-3}	2000
Specific heat capacity	c_{pH}	J.(kg. K)$^{-1}$	1842
Temperature of unaffected rock	T_H	°C	200
Thermal conductivity	λ_H	W.(m. K)$^{-1}$	2,5
Depth of exchanger	h	m	3000

The heat transport medium (heat carrier) is considered water in liquid phase. (Tab. 2).

Tab 2 Physical properties of H_2O

Density	ρ	$kg.m^{-3}$	983,2
Specific heat capacity	c_p	$J.(kg.\ K)^{-1}$	4185
Temperature	t_0	°C	40
Thermal conductivity	λ	$W.(m.\ K)^{-1}$	0,651
Conductivity Heat Transfer coefficient	α	$W.\ m^{-2}.\ K^{-1}$	5000

METHOD 1 – GAUSS DIVERGENCE INTEGRAL

The rock will be considered as half-limited massive influenced by sudden subcooling. The limiting condition is that the final temperature is equal to zero (Θ = 0) at the time τ = 0s on the pipe surface x = 0m.

Therefore, in the solution of the differential equation for non-steady heat transfer [2]:

$$T(x,\tau) - T_S = \Theta(x,\tau) = \sum_{i=1}^{\infty}\left[A(\varepsilon_i)\cos(\varepsilon_i x) + B(\varepsilon_i)\sin(\varepsilon_i x)\right]e^{-\varepsilon_i{}^2 a\tau} \tag{1}$$

it is necessary to put the coefficient $A(\varepsilon_i)$ as equal to zero and consider the solution in the following simplified expression:

$$\Theta(x,\tau) = \sum_{i=1}^{\infty} B(\varepsilon_i)\sin(\varepsilon_i x)e^{-\varepsilon_i{}^2 a\tau} \tag{2}$$

After several modifications, we can reach a relationship for the calculation of the heat flow as a function of time

$$q_S = (T_S - T_a)\sqrt{\frac{\lambda_H \cdot \rho_H \cdot c_H}{\pi.\tau}} \qquad \mathrm{W/m^2} \qquad (3)$$

If we input a time step of 10 h, the specific heat flow after the first ten hours will be [3]:

$$q_S = (200 - 40)\sqrt{\frac{2{,}5.2000.1842}{\pi.36000}} \qquad = 1443{,}85 \qquad \mathrm{W/m^2} \qquad (3)$$

This method considers thermal accumulation potential instead of increasing thermal resistance. Thermal accumulation potential has an equivalent relationship with the convection coefficient of heat transfer on the flowing heat carrier side.

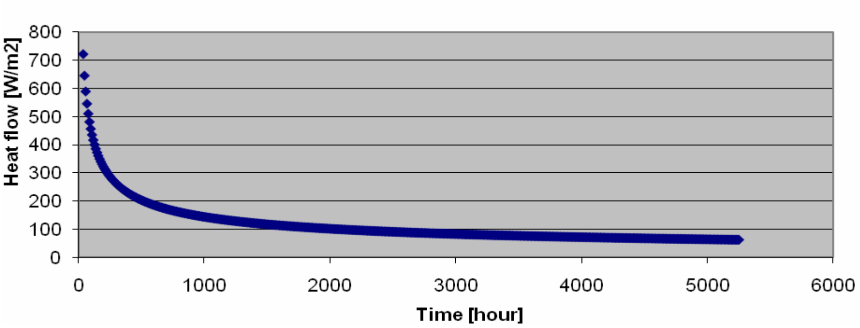

Fig. 1 Heat flow from the rock massive into heat carrying medium.

As can be seen from the graph above, the specific heat flow will be stabilized at a value of 60 W.m^{-2}

The constant temperature fields in a continually subcooled rock massive are determined by using of Gauss divergence integral, where the beginning of a process is given [3]:

$$\frac{x}{2\sqrt{a.\tau}} = 0 \qquad \psi\left(\frac{x}{2\sqrt{a.\tau}}\right) = 0 \tag{4}$$

And for the unfinite time:

$$\frac{x}{2\sqrt{a.\tau}} \to \infty \qquad \psi\left(\frac{x}{2\sqrt{a.\tau}}\right) \to 1 \tag{5}$$

Then the temperature at time τ and at a diameter distance r from the axis of the pipe will be:

$$\frac{T_a - T(\tau,t)}{T_a - T_S} = 1 - \psi\left(\frac{x}{2\sqrt{a.\tau}}\right) \Rightarrow \qquad T(\tau,r) = T_a - \left(1 - \psi\left(\frac{x}{2\sqrt{a.\tau}}\right)\right).(T_a - T_S) \qquad [°C] \tag{6}$$

Where

T_a is the temperature of rock massive in the unaffected area

T_S is the temperature of the heat carrying medium (i.e. water) at the point of inlet into the heat exchanger pipe

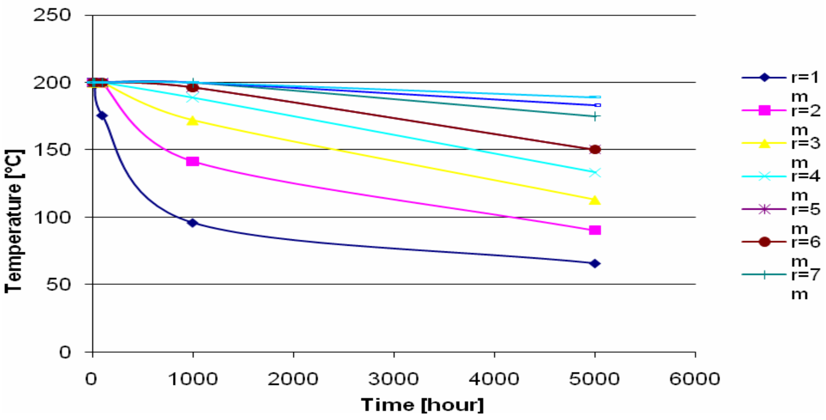

Fig. 2 Hot rock massive temperature drop depending on time.

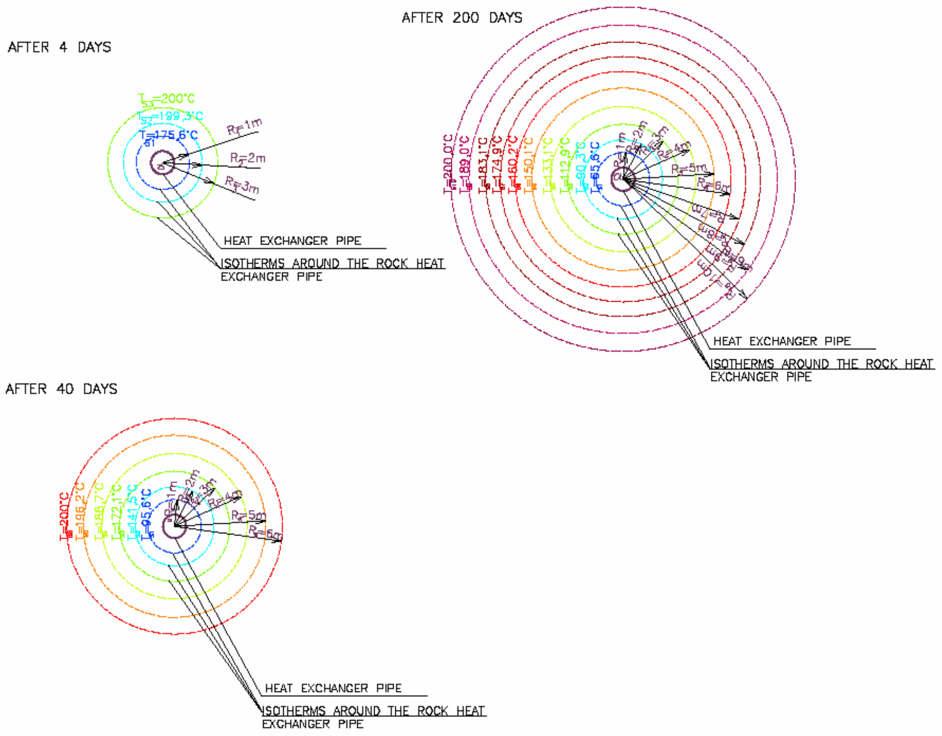

Fig. 3 Fields of constant temperatures in gradually sub cooled rock massive.

The calculation was performed using the MATHEMATICA program and its accuracy was verified experimentally in laboratory tests at the Institute of Thermal Power Engineering, STU. The experiments were carried out in an experimental modelling stand with smaller diameter, thus the verification of the above described calculation could be confirmed only for smaller diameters of solid rock material.

In the experiment, water at 80°C was used as a heat source for solid rock. The cooling medium, simulating the role of the heat carrying medium in the underground heat exchanger was water at 17°C.

The comparison of the above mentioned calculation and experimental results are shown in fig. 4.

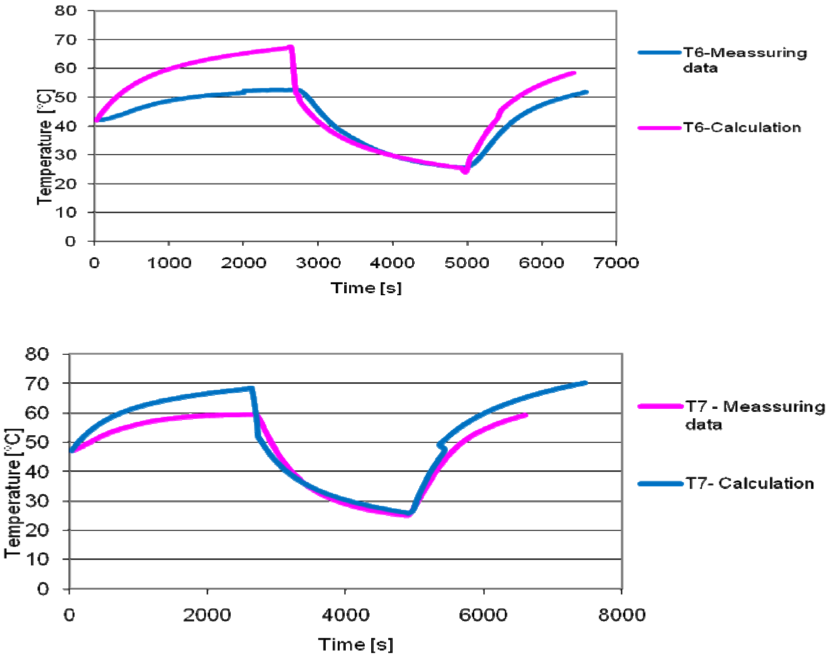

Fig. 4 Comparison of experiment and calculation (T6: R=0,5m, T7:R=1m).

METHOD 2 – FORCHHEIMER METHOD

Due the fact that an experimental verification of the thermo kinetic behavior for sub cooled rock massive can be carried out only for limited diameters, we used another calculation method to verify the calculated results. Both calculation methods, the Gauss divergence integral and the Forchheimer, method were compared.

For the calculation using the Forchheimer method, the same limiting conditions, i.e. unaffected rock massive temperature of 200°C (T_H) and temperature of the heat carrying medium of 40°C.(Ts) were expected
A horizontal pipe with diameter r = 0,21m was considered as geometrical givens. The surface temperature of the heat exchanger pipe was taken as equal to the water temperature, i.e. 40°C.

In the literature [4] the heat flow density is:

$$q_{hor} = \frac{\pi\left(T_H - T_S\right)}{\frac{1}{2.\lambda_{Hor}} \ln\left(\frac{2h}{d} + \sqrt{\left(\frac{2h}{d}\right)^2 - 1}\right)} \qquad [\mathrm{W.m^{-1}}] \qquad (7)$$

The curvature of isotherms is expressed in the numerator of the equation, and heat resistance is expressed in the denominator of the fraction, where λ_{Hor} is thermal conductivity of the rock, d is the diameter of the pipe and h is depth of exchanger.
Thus, the heat flow density at the inlet into the heat exchanger pipe (water 40 °C, rock 200 °C) is:

$$q_{hor} = \frac{\pi(200-40)}{\frac{1}{2.2{,}5}\ln\left(\frac{2.500}{0{,}21}+\sqrt{\left(\frac{2.500}{0{,}21}\right)^2-1}\right)} = \qquad 43{,}85 \qquad [\mathrm{W.m^{-1}}]$$

Of course, the heat flow density depends on the temperature difference between the water and the rock, and therefore is not constant along the whole pipe.

COMPARISON OF RESULTS

The results from both calculations methods were compared. After re-calculation of the first method, the Gauss divergence integral for a pipe of diameter d = 0,21m and at different temperature gradients, we obtained the following result values:

Tab 3 Comparison of results

Mean media temperature [°C]	Temperature gradient [K]	Heat flow [W/m]	
		2.method	1.method
40	160	43,8703	42,92694
50	150	41,1284	40,25637
60	140	38,3865	37,57789
70	130	35,6446	34,88226
80	120	32,9027	32,20971
90	110	30,1608	29,52134
100	100	27,4189	26,84154
110	90	24,6770	24,15778
120	80	21,9352	21,47006
130	70	19,1933	18,78908
140	60	16,4514	16,10255
150	50	13,7095	13,42077
160	40	10,9676	10,73503
170	30	8,2257	8,051274
180	20	5,4838	5,368307
190	10	2,7419	2,683758

CONCLUSION

As we can see in the Table 3, both calculating methods gave similar results. The correlation between calculated values and the limited experimental results is also satisfactory, therefore both presented methods for the thermo kinetic behavoiur of solid rock massive and heat underground heat exchanger can be considered as suitable bases for the design calculation of geothermal Hot-Dry-Rock power systems.

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