NON-CONVENTIONAL PRODUCTION MACHINES

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ABSTRACT

The paper deals with machines employing parallel-kinematics structures (PKS). They represent a relatively new generation of machine tools. Depending on the number of struts, the machines are referred to as hexapod or tripod machines. Such machines offer several advantages compared to conventional machine tools with serial kinematics, such as high flexibility, high stiffness, and high accuracy. They are very suitable for High Speed-Machining (HSM), light machining and have attracted a wide interest in the manufacturing industry. In order to achieve a desired positioning accuracy and stability, the static and dynamic properties of the machine must be researched and mathematically described. The calculation of the estimate of positioning deviation, including the respective uncertainty and covariances, is a much more complicated task compared to serial kinematics.
Keywords: parallel kinematic structures, positioning accuracy, measurement uncertainty

INTRODUCTION

Early in the nineties, the development of the machine began to build on parallel kinematics structures whose movement mechanism consists of several parallel aligned movement parts. Parallel kinematic structures (hereafter PKS) requires a different approach to managing the movement of the tool head, compared to a machine with a serial kinematic structure. It is difficult to create a mathematical model that incorporates all the effects of kinematic and dynamic behavior. The calculation of influences is not simple because the movement member are arranged in parallel. The keystone for mathematical model are motion equations that describe motion of telescopic rods in motion of end-point of tool.

TRICEPT

The tricept has a specific kinematic configuration and subassemblies. Therefore special definitions of its single parts are needed.

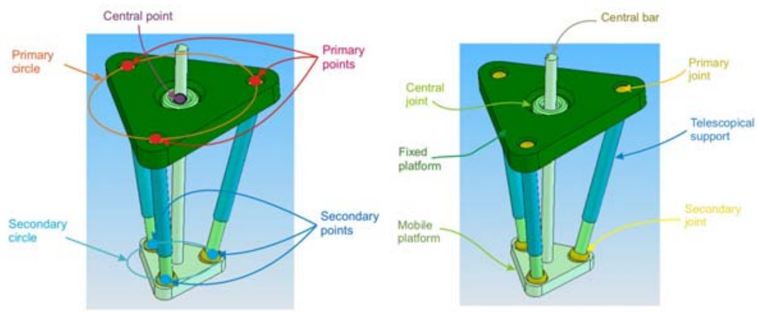

Fig. 1 Basic design of tricept.

The structure of the Tricept consists of three identical parallel bars, in this case two telescopic rods and passive central rod. These bars connect the fixed and the movable platform. With the joints fixing these bars to the platforms there is a slewing of the movable platform, on which the tool is located.
The base of the tricept is a stable platform on which the primary joints and the central joint are located. A telescopic rod is connected to a stable platform with the primary joint. At the opposite end of the telescopic rod is a rod with the secondary joint connected to the carrier. The carrier is fixed to the central rod. The axis of the central rod always passes through the central point. The carrier holds the tool or technology head. The whole Tricep is in our case placed vertically on the structure.

WORKSPACE

The shape of the PKS workspaces are markedly different, and a separate analysis is required for analyzing their shape and quantitative parameters for each device and varying degrees of difficulty.

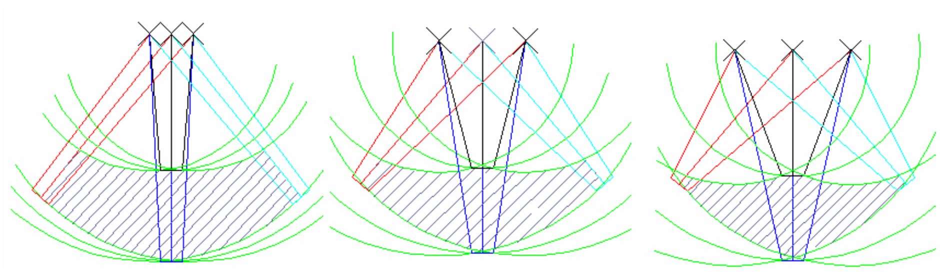

Fig. 2 Extreme positions of telescopic rod.

Four positions of the tricept, which are shown in black, blue, red and turquoise lines are schematically represented in Fig 2. The black position shows the situation when both the telescopic rods are retracted and blue shows the position when all the telescopic rods are extended. Red and turquoise represent alternative extreme positions in which the opposite pole and extended angle is in the range 0 ° - 40 °. The green line shows the minimum and maximum range of extension of each telescopic rod. The shaded area represents the working section.
Examination of the workspace of the tricept confirms the difference of shape of symmetry of the joint seat on the secondary circle and symmetry of joint in which the central rod moves. In the first case, the symmetry is identical to the symmetry of an equilateral triangle, while for the central joint it is mirror symmetry. This mirror symmetry also has a workspace.

Fig. 3 Shape of workspace.

THE MOTION EQUATION OF TRICEPT

To achieve the desired effector position it is necessary to calculate and then set all the required lengths of the telescopic rods. In order to create a mathematical model of the control of the sliding telescopic rods we use vector loops In the process of creating a mathematical model of control for the sliding telescopic rods we progress by analysing the vector loops.

The Tricep with the vector loop is illustrated in Fig. 4. The loop composed of vectors $\overrightarrow{SP}$, $\overrightarrow{PL}$, $\overrightarrow{LB}$, $\overrightarrow{SB}$ describes the position of point *L*, which is important for the calculation of vector $\overrightarrow{LB}$ and of which length is identical to length of the extended rod.

Calculation of coordinates of point *L* is done by procedure, for which is necessary to know the coordinates of point *P*. Calculation of the coordinates of a point P is illustrated in Fig. 3 left. The location of the central rod is defined by two points - S and Q. The Tricep is placed in a coordinate system so that point S was in the beginning. The second point that defines the position of the central bar is point Q. It is a curve point, followed by the moving end point of the instrument and also includes the instrument. When moving, there is a change in the distance of points S and Q, this means that the central rod is inserted into the central joint, but a central part of the joint from the kinematic point of view is important.

The first step in creating equations of movement is to determine the coordinates of point P. Because it lies on the junction of points *S* and *Q*, the first step derives from the fact that vectors $\overrightarrow{SP}$ and $\overrightarrow{SQ}$ always have the same value of cosin. We will obtain the cosins of vector $\overrightarrow{SQ}$ by entering the coordinates of point *Q* as follows:

$$\cos\alpha = \frac{Q_x}{\sqrt{{Q_x}^2 + {Q_y}^2 + {Q_z}^2}}; \tag{1}$$

$$\cos\beta = \frac{Q_y}{\sqrt{{Q_x}^2 + {Q_y}^2 + {Q_z}^2}}; \tag{2}$$

$$\cos\gamma = \frac{Q_z}{\sqrt{{Q_x}^2 + {Q_y}^2 + {Q_z}^2}}; \tag{3}$$

The distance between points Q and P is constant. This distance has to be applied to all three coordinates. The exact coordinates of each component we can create by direction cosine and then add them to the coordinates of point Q. The results are the explicitly determined coordinates of point P.

$$P_x = Q_x + QP_x = Q_x + |QP| \cdot \cos\alpha \tag{4}$$

$$P_y = Q_y + QP_y = Q_y + |QP| \cdot \cos\beta \tag{5}$$

$$P_z = Q_z + QP_z = Q_z + |QP| \cdot \cos\chi \tag{6}$$

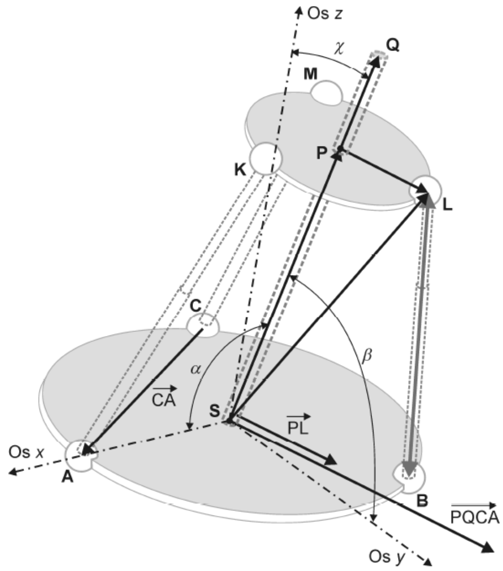

Fig. 4 Loop of tricept's movement equations.

The second step is to calculate the coordinates of point L. The calculation will use analytical geometry. Vector $\overrightarrow{PQ}$ is connected to vector $\overrightarrow{SQ}$ and in this way changes direction, which is necessary for a description of the change of position of vector $\overrightarrow{PL}$. The direction and orientation of searched vector $\overrightarrow{PL}$ can be obtained by vector multiplying of vectors $\overrightarrow{PQ}$ and $\overrightarrow{CA}$.

$$\overrightarrow{PQ} x \overrightarrow{CA} = \overrightarrow{PQCA} \tag{7}$$

In this way vector $\overrightarrow{PQCA}$ is obtained, which has the same direction and orientation as the desired vector $\overrightarrow{PL}$, however its length is different. It is necessary to change the length of this vector by multiplying by desirable constant *k*. Thus we can obtain the vector with the correct direction, orientation and length.

$$\overrightarrow{PQCA} \cdot k_{PQCA} = \overrightarrow{PL} \tag{8}$$

If we add vector $\overrightarrow{PL}$ to vector $\overrightarrow{SP}$ we will obtain vector $\overrightarrow{SL}$, thus also the correct coordinates of point L.

$$\overrightarrow{SP} + \overrightarrow{PL} = \overrightarrow{SL} \tag{9}$$

Itemized equation for calculating the length of the telescopic rod is:

$$\left|\overrightarrow{BL}\right| = \sqrt{\begin{aligned}&\left[\left\{\left[Q_x - |QP|\cdot\cos\alpha\right] + \left(\left[\left(\left(|QP|\cdot\cos\beta\right)\cdot CA_z\right) - \left(\left(|QP|\cdot\cos\chi\right)\cdot CA_y\right)\right]\cdot k_{PQCA}\cdot i\right)\right\} - B_x\right]^2 + \\ &\left[\left\{\left[Q_y - |QP|\cdot\cos\beta\right] + \left(\left[\left(\left(|QP|\cdot\cos\chi\right)\cdot CA_x\right) - \left(\left(|QP|\cdot\cos\alpha\right)\cdot CA_z\right)\right]\cdot k_{PQCA}\cdot j\right)\right\} - B_y\right]^2 + \\ &\left[\left\{\left[Q_z - |QP|\cdot\cos\chi\right] + \left(\left[\left(\left(|QP|\cdot\cos\alpha\right)\cdot CA_z\right) - \left(\left(|QP|\cdot\cos\beta\right)\cdot CA_x\right)\right]\cdot k_{PQCA}\cdot k\right)\right\} - B_z\right]^2\end{aligned}} \qquad (10)$$

This procedure shows the calculation of the length of the telescopic rods. Calculation of the lengths of the other rods is by analogy. It must be noted that to achieve the desired end of the effector position, i.e. the required value of point Q, the function involving the calculation of the length of telescopic rods must be found.

THE PRECISION OF POSITIONING

The positioning accuracy of any manufacturing facility is reflected by the closeness of the correlation between the actual point reached by the final effector position, and that which was programmed through the control system. A comparison of the conventional serial kinematics with parallel kinematics, shows a few key differences. The precision positioning of device with parallel kinematics is much more complicated and is more demanding than with serial kinematics. Manufacturing tolerances, installation errors and displacement of individual connections cause variations over the nominal kinematic parameters of the system. As a result, when the wrong nominal values of these parameters are used in the control system equipment, the resulting effector position does not correspond to the desired value.

Effector positioning accuracy depends on the geometry errors, compliance and time-variable temperature. Geometric errors are caused by inaccuracies in manufacturing, improper location of individual components or wear of joints. The errors in positioning have their origin in the flexibility of joints and parts which change shape due their own weight and external load. They also depend on the actual position of the effector.

Thermal errors owe their origin to thermal stresses and the expansion of materials due to heat generated by internal and external sources, such as. motors, bearings and due to changes in ambient temperature. The flexibility of individual components and their connections has a significant impact on the functioning of the equipment and its stability.

UNCERTAINTY ESTIMATION FOR POSITIONING ENDPOINT EFFECTOR

In calculating the estimated positioning accuracy, a key issue is determining the function that describes the positioning of the effector (point Q), depending on the extended telescopic rods. Extending the telescopic rod is, in fact, the only possibility of influencing the position of point Q. Unlike serial kinematics, positioning leads to complicated trigonometric functions containing members, which then lead to nonlinear solutions. The practical consequence is that the positioning accuracy depends not only on the accuracy of the extended telescopic poles, but also the position of Q in the workspace tricept.

For documentation on the difficulty in calculating the uncertainty of achieving the desired position of point Q, see the list of geometric parameters that their work contributes to the overall uncertainty of achieving the desired position:

a) the relative position of the joint center on the base plate, thus the distance of the points AS, BS, CS,

b) the relative position of the joints on the base plate, thus the distance of points CA, CB, BA,

c) the relative position of points towards the center of the movable plate, thus the distance of points KP, MP, LP,

d) the relative position of the joint center on the movable plate, thus the distance of the points KM, KL, ML,
e) the distance of the immovable and movable plates on the center rod, thus the distance of points SP,
f) the distance of the end-points of the effector and of the point of attachment of the movable plate on the center rod, thus the distance of points PQ,
g) the length of the telescopic rods, thus the distance of points KA, MC, LB.

It is clear that the nominal value of those parameters affect the geometry defects, errors or tractable thermal errors. If we identify f_O as the function for determining the position of point Q, depending on the parameters AS, BS and LB in the previous list, namely

Q = f_Q(AS, BS, CS, CA, CB, BA, KP, MP, LP, KM, KL, ML, SP, PQ, KA, MC, LB)

The uncertainty u_O of calculation of the position of point Q is given by the law governing the spread of uncertainty, which in this case means

$$u_Q^2 = \left(\frac{\partial f_Q}{\partial \mathrm{AS}}\right)^2 u_{\mathrm{AS}}^2 + \left(\frac{\partial f_Q}{\partial \mathrm{BS}}\right)^2 u_{\mathrm{BS}}^2 + \ldots + \left(\frac{\partial f_Q}{\partial \mathrm{L}B}\right)^2 u_{\mathrm{LB}}^2 + \left(\frac{\partial f_Q}{\partial \mathrm{AS}}\right)\left(\frac{\partial f_Q}{\partial \mathrm{BS}}\right) u_{\mathrm{ASBS}} + \ldots \left(\frac{\partial f_Q}{\partial \mathrm{M}C}\right)\left(\frac{\partial f_Q}{\partial \mathrm{L}B}\right) u_{\mathrm{MCLB}}$$

where

AS, BS, .. LB are relevant parameters,
u_{AS}, u_{BS}, ..., u_{LB} are values of uncertainties of various parameters ,
u_{ASBS}, ..., u_{MCLB} are values of covariances between relevant parameters.

We can see therefore that we find the analytical expressions of partial derivatives function f_O under different parameters. Uncertainties of the parameters of the u_{AS}, u_{BS}, ..., u_{LB} are composed of various sub-sources, as described above.

CONCLUSION

The contribution aims to demonstrate a method of creating mathematical equations through which the telescope is controlled by slider bars. The equations are developed using analytical geometry. Another type of PKS requires a similar mathematical approach, but must be based on the kinematic characteristics of the type of PKS.
The article also outlined the theoretical difficulties in determining the positioning accuracy endpoint effector of a parallel kinematic structure – the tricept. Based on the analysis of the vector loop example, we reported the determination of dependence between extending a telescopic rod and effector location. On the basis of the law of the spread of the uncertainty we point out some problems in the analytical formulation of theoretically achievable positioning accuracy.
Note that by employing the mentioned method we can simulate the theoretically achievable values of uncertainties of the determination of the position of the end point of effector Q. This means that on the basis of an analysis of the tricept structure, its geometry, allowable construction tolerances, properties of the telescopic rods etc, we can define the theoretical ability of achieving the required position without external load. The actual deviation of the effector from desired position in the workspace of tricept have to be determined by the measurement and then adjusted by corrections obtained from the estimation theory model. But this is a much more complicated theoretical task.

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