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Statistical interpretation of data —

Part 6:
Determination of statistical tolerance intervals

Interprétation statistique des données —

Partie 6: Détermination des intervalles statistiques de dispersion

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Annex F
(informative)

Computation of factors for two-sided parametric statistical tolerance intervals

In the field of mathematical statistics, the interval for the case of unknown mean μ and unknown standard deviation σ is called a p-content tolerance interval with confidence level $1 - \alpha$ for a normal distribution. The symbol β is sometimes used instead of the symbol p. Although the definition of a p-content tolerance interval is simple, the computation of precise values of tolerance factors is fairly difficult, particularly without the use of a computer. We consider the tolerance interval constructed by $[\bar{x} - k \times s, \bar{x} + k \times s]$, where $\bar{x}$ and s are, respectively, the sample mean and the sample standard deviation.

The value of a tolerance factor is the solution in k of the following integral equation

$$\sqrt{\frac{n}{2\pi}} \int_{-\infty}^{\infty} F(x,k)\, e^{-\frac{nx^2}{2}}\, dx - 1 + \alpha = 0, \qquad \text{(F.1)}$$

where

$$F(x,k) = \int_{\frac{f R^2(x)}{k^2}}^{\infty} \frac{t^{\frac{f}{2}-1} e^{-\frac{t}{2}}}{2^{\frac{f}{2}} \Gamma\left(\frac{f}{2}\right)} dt,$$

and $R(x)$ is the solution of the equation $\Phi(x+R) - \Phi(x-R) - p = 0$.

In the formula for $F(x,k)$, Formula (F.1), the symbol f is the number of degrees of freedom, which depends on the number of samples and the number of observations in each sample.

NOTE 1 For one sample of size n the degrees of freedom is $f = n - 1$.

NOTE 2 For m samples of the same size n (balanced model) the degrees of freedom is $f = m(n - 1)$.

NOTE 3 For m samples of sizes $n_1, n_2, \ldots, n_m$ (unbalanced model) the degrees of freedom is

$$f = \sum_{i=1}^{m} (n_i - 1) = \left(\sum_{i=1}^{m} n_i \right) - m.$$

In this case Formula (F.1) is modified; n is substituted by n_i and k by k_i and we obtain separate solutions k_i for each sample.

Analytical derivation of the solution of Formula (F.1) with respect to k is difficult, if not impossible, so approximate methods for the computation of factor k have been used in the past. In the previous standard of tolerance interval (ISO 3207:1975), the factors in the table for two-sided statistical tolerance interval for the case of unknown μ and σ were obtained by such a method.

More recently, computer programs that use numerical integration for the exact computation of the factors have been developed. In Annex D the factors, which were derived by an iterative process using numerical integration, have been calculated to give at least the required confidence level.

Extensive tables of the factor k for two-sided statistical tolerance interval for the normal distribution with unknown μ and σ have been published by Garaj and Janiga[9] with an introduction to the tables given in English, French, German, and Slovak. These tables correspond to the column m=1 of the tables of Annex D in this part of ISO 16269, but the number of entries and the ranges of n, p and α are larger in the tables by Garaj and Janiga[9].

In Annex D are given tables of the factor k for two-sided statistical tolerance intervals for the normal distributions with unknown μ_i(i = 1,2,...,m; m = 2(1)10) and unknown common σ.

Extensive tables of the factor k for two-sided statistical tolerance interval for the normal distribution with unknown μ and common σ have also been published by Garaj and Janiga[10] with an introduction to the tables given in English, French, German, and Slovak. These tables correspond to the columns m=2(1)10 of the tables in Annex D in this part of ISO 16269, but the number of entries, the number of decimal places and the ranges of m, n, p and α are larger in the tables by Garaj and Janiga.[10]

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